Lecture 1

GNSS measurements and their combinations

Contact: jaume.sanz@upc.edu
Web site: http://www.gage.upc.edu
Authorship statement

The authorship of this material and the Intellectual Property Rights are owned by J. Sanz Subirana and J.M. Juan Zornoza.

These slides can be obtained either from the server http://www.gage.upc.edu, or jaume.sanz@upc.edu. Any partial reproduction should be previously authorized by the authors, clearly referring to the slides used.

This authorship statement must be kept intact and unchanged at all times.

5 March 2017
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
GPS SIGNAL STRUCTURE

Two carriers in L-band:
- $L_1 = 154 \text{ fo} = 1575.42 \text{ MHz}$
- $L_2 = 120 \text{ fo} = 1227.60 \text{ MHz}$
  where $\text{fo} = 10.23 \text{ MHz}$

- C/A-code for civilian users $[X_C(t)]$
- P-code only for military and authorized users $[X_P(t)]$
- Navigation message with satellite ephemeris and clock corrections $[D(t)]$

\[ S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t) D^{(k)}(t) \sin(\omega_1 t + \phi_{L_1}) + a_C X_C^{(k)}(t) D^{(k)}(t) \cos(\omega_1 t + \phi_{L_1}) \]
\[ S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t) D^{(k)}(t) \sin(\omega_2 t + \phi_{L_2}) \]
GPS Code Pseudorange Measurements

\[ S_{L_1}^{(k)}(t) = a_P X_P^{(k)}(t) D^{(k)}(t) \sin(\omega_t + \varphi_{L_1}) + a_C X_C^{(k)}(t) D^{(k)}(t) \cos(\omega_t + \varphi_{L_1}) \]

\[ S_{L_2}^{(k)}(t) = b_P X_P^{(k)}(t) D^{(k)}(t) \sin(\omega_t + \varphi_{L_2}) \]

\[ P(T) = c \Delta T = c \left[ t_{rec}(T) - t_{sat}(T - \Delta T) \right] \]

From hereafter we will call:
- **C₁** pseudorange computed from \( X_C(t) \) binary code (on frequency 1)
- **P₁** pseudorange computed from \( X_P(t) \) binary code (on frequency 1)
- **P₂** pseudorange computed from \( X_P(t) \) binary code (on frequency 2)
GPS Carrier Phase Measurements

\[ S_{L_1}^{(k)}(t) = a_p X_P^{(k)}(t) D^{(k)}(t) \sin(\omega t + \varphi_{L_1}) + a_c X_C^{(k)}(t) D^{(k)}(t) \cos(\omega t + \varphi_{L_1}) \]

\[ S_{L_2}^{(k)}(t) = b_p X_P^{(k)}(t) D^{(k)}(t) \sin(\omega t + \varphi_{L_2}) \]

**Carrier phase**

**Carrier beat phase:**

\[ \phi_L(T) = \phi_{L \text{rec}}(T) - \phi_{L \text{sat}}(T - \Delta T) \]

\[ = \frac{c}{\lambda} \Delta T + N \]

**Unknown ambiguity**

From hereafter we will call:

- \( L_1 = \lambda_1 \phi_{L_1} \) measur. computed from the carrier phase on frequency 1
- \( L_2 = \lambda_2 \phi_{L_2} \) measur. computed from the carrier phase on frequency 2

- \( C_1 \) pseudorange computed from \( X_C(t) \) binary code (on frequency 1)
- \( P_1 \) pseudorange computed from \( X_P(t) \) binary code (on frequency 1)
- \( P_2 \) pseudorange computed from \( X_P(t) \) binary code (on frequency 2)
Carrier and Code pseudorange measurements

$P_1 = c\Delta T = c [t_{rec}(T) - t_{sat}(T - \Delta T)]$

$P_1 \approx \rho + \text{clock offset} \approx 20.000\text{Km}$

$P_1$ is basically the geometric range ($\rho$) between satellite and receiver, plus the relative clock offset. The range varies in time due to the satellite motion relative to the receiver.

$P_1$ is an absolute measurement (unambiguous)
Phase and Code pseudorange measurements

Each time that the receiver lose the phase lock, the unknown ambiguity changes by an integer number of $\lambda$.

$L_1(T) = c \Delta \tilde{T} + \lambda_1 N_1$

$\rho \approx \rho + \text{clock offset} + \lambda_1 N_1$
Code and Carrier Phase measurements

Code (unambiguous but noisier)

Carrier Phase (ambiguous but precise)

Ambiguity
### GPS measurements: Code and Carrier Phase

#### Antispoofing (A/S):
The code P is encrypted to Y. 
- Only the code C at frequency L1 is available.

<table>
<thead>
<tr>
<th>Code measurements</th>
<th>Phase measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C₁</strong></td>
<td><strong>L₁</strong></td>
</tr>
<tr>
<td>300 m</td>
<td>19.05 cm</td>
</tr>
<tr>
<td>3 m</td>
<td>2 mm</td>
</tr>
<tr>
<td></td>
<td>Precise but ambiguous</td>
</tr>
<tr>
<td><strong>P₁ (Y₁): encrypted</strong></td>
<td><strong>L₂</strong></td>
</tr>
<tr>
<td>30 m</td>
<td>24.45 cm</td>
</tr>
<tr>
<td>30 cm</td>
<td>2 mm</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>P₂ (Y₂): encrypted</strong></td>
<td></td>
</tr>
<tr>
<td>30 m</td>
<td></td>
</tr>
<tr>
<td>30 cm</td>
<td></td>
</tr>
</tbody>
</table>

- **Wavelength (chip-length):**
  - C₁: 300 m
  - P₁ (Y₁): 30 m
  - P₂ (Y₂): 30 m

- **σ noise (1% of λ):**
  - C₁: 3 m
  - P₁ (Y₁): 30 cm
  - P₂ (Y₂): 30 cm

- **Main characteristics:**
  - Unambiguous but noisier
  - Precise but ambiguous

[*] The codes can be smoothed with the phases in order to reduce noise (i.e., C₁ smoothed with L₁ → 50 cm noise)
RINEX FILES
Pseudoranges (C/A, P1, P2),
phase tracking (L1, L2)
Navigation data (D(t))

One or multiple antennas

Low-noise Amplifiers

Low-noise Amplifiers

RF/IF & SAMPLING

DLL Tracking RCVR Demodulator

Navigation data processing
&
Pseudorange correction

Kalman filter position estimation

DISPLAY

Man-Machine Interface

Aiding or integrated receiver

EXTERNAL SENSORS

Master of Science in GNSS
@ J. Sanz & J.M. Juan
GNSS Format Descriptions

- GNSS data files follow a well defined set of standards formats: RINEX, ANTEX, SINEX...
- Understanding a format description is a tough task.
- These standards are explained in a very easy and friendly way through a set of html files.
- Described formats:
  - Observation RINEX
  - Navigation RINEX
  - RINEX CLOCKS
  - SP3 Version C
  - ANTEX

Open GNSS Formats with Firefox internet browser

More details at: http://www.gage.upc.edu/gLAB
## RINEX measurement file

### HEADER

```
RINEX VERSION / TYPE 2
PGM / RUN BY / DATE RGRINEX0 V2.4.1 UX AUSLIG 10-JAN-97 10:19
Australian Regional GPS Network (ARGN) - COCOS ISLAND
BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER "AS" CONDITION
MARKER NAME COCO
MARKER NUMBER AU18
OBSERVER / AGENCY mrg
OBSESSION SNR-8100 93.05.25 / 2.8.33.2
REC # / TYPE / VERS 126
ANT # / TYPE 327
APPROX POSITION XYZ
ANTENNA: DELTA H/E/N
WAVELENGTH FACT L1/2
# / TYPES OF OBSERV
SNR is mapped to signal strength [0,1,4-9]
TIME OF FIRST OBS
TIME OF LAST OBS
```

### MEASUREMENTS

```
<table>
<thead>
<tr>
<th>T</th>
<th>L1</th>
<th>L2</th>
<th>P1</th>
<th>L1</th>
<th>L2</th>
<th>P1</th>
<th>L1</th>
<th>L2</th>
<th>P1</th>
<th>L1</th>
<th>L2</th>
<th>P1</th>
<th>L1</th>
<th>L2</th>
<th>P1</th>
</tr>
</thead>
<tbody>
<tr>
<td>97</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>7</td>
<td>30</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>25</td>
<td>9</td>
<td>5</td>
<td>23</td>
<td>17</td>
<td>6</td>
</tr>
<tr>
<td>22127</td>
<td>1118981.28445</td>
<td>22127685.4014</td>
<td>&lt;=</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22672</td>
<td>8969469.30045</td>
<td>22672158.5184</td>
<td>&lt;=</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22594902.367</td>
<td>-12949753.8257</td>
<td>-10090708.5394</td>
<td>&lt;=</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22731128.796</td>
<td>-1162184.9517</td>
<td>-9055464.1694</td>
<td>&lt;=</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24610920.702</td>
<td>-924108.1746</td>
<td>-720085.6704</td>
<td>&lt;=</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20718775.074</td>
<td>-18605935.4749</td>
<td>-14498133.9734</td>
<td>&lt;=</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20842713.610</td>
<td>-19083282.8929</td>
<td>-14870090.5546</td>
<td>&lt;=</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

---

**Note:** The RINEX file contains measurements from a GPS receiver, specifically from the Australian Regional GPS Network (ARGN) at COCOS ISLAND. The file includes metadata such as the RINEX version, observer details, and measurement data. The data is structured with columns for time, signal strength, and other parameters, typical of GPS data records.
# RINEX measurement file

<table>
<thead>
<tr>
<th>OBSERVATION DATA</th>
<th>G (GPS)</th>
<th>RINEX VERSION / TYPE</th>
<th>PGM / RUN BY / DATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGRINEX0 V2.4.1 UX AUSSIE</td>
<td>10-JAN-97 10:19</td>
<td>COMMENT</td>
<td>COMMENT</td>
</tr>
<tr>
<td>Australian Regional GPS Network (Argn) - Cocos Island</td>
<td>BIT 2 OF LLI (+4) FLAGS DATA COLLECTED UNDER &quot;AS&quot; CONDITION</td>
<td>COMMENT</td>
<td>COMMENT</td>
</tr>
<tr>
<td>-0.00000000000103</td>
<td>HARDWARE CALIBRATION (S)</td>
<td>COMMENT</td>
<td>COMMENT</td>
</tr>
<tr>
<td>-0.00000000054663</td>
<td>CLOCK OFFSET (S)</td>
<td>COMMENT</td>
<td>COMMENT</td>
</tr>
<tr>
<td>COCO</td>
<td>MARKER NAME</td>
<td>MARKER NUMBER</td>
<td>OBSERVER / AGENCY</td>
</tr>
<tr>
<td>AU18</td>
<td>ROGUE SNR-8100</td>
<td>93.05.25 / 2.8.33.2</td>
<td>REC # / TYPE / VERS</td>
</tr>
<tr>
<td>126</td>
<td>DORNE MARGOIN T.</td>
<td>741950.3241 6190961.9624 -1337769.9813</td>
<td>ANT # / TYPE</td>
</tr>
<tr>
<td>SNR is mapped to signal strength [0,1,4-9]</td>
<td>INTERVAL: DELTA X/Y/Z</td>
<td>WAVELENGTH FACT L1/L2</td>
<td># / TYPES OF OBSERV</td>
</tr>
<tr>
<td>SNR: &gt;500 &gt;100 &gt;50 &gt;10 &gt;5 &gt;0</td>
<td>COMMENT</td>
<td>COMMENT</td>
<td>COMMENT</td>
</tr>
<tr>
<td>sig: 9 8 7 6 5 4 1</td>
<td>COMMENT</td>
<td>COMMENT</td>
<td>COMMENT</td>
</tr>
<tr>
<td>1997 1 9 0 7 30.00000000</td>
<td>TIME OF FIRST OBS</td>
<td>TIME OF LAST OBS</td>
<td>END OF fichier</td>
</tr>
<tr>
<td>1997 1 9 23 59 30.00000000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 97 1 9 0 7 30.00000000 | 87 | 1 25 9 5 23 17 6 |
| 22127685.105 | -14268715.899 | 8 | -11118481.28445 | 22127685.4014 | <=1 | 1 |
| 22672158.746 | -11510817.892 | 7 | -8969469.30045 | 22672158.5184 | <=25 | 25 |
| 22594992.367 | -12949753.825 | 7 | -10090708.53945 | 22594992.7394 | <=9 | 9 |
| 22731128.796 | -11621184.951 | 7 | -9055464.16945 | 22731128.0094 | <=5 | 5 |
| 24610920.702 | -9241081.174 | 6 | -720085.67045 | 24610920.0404 | <=23 | 23 |
| 20718775.074 | -18605935.474 | 9 | -14498133.97346 | 20718775.6074 | <=17 | 17 |
| 20842713.610 | -19083282.892 | 9 | -14870090.55546 | 20842713.4814 | <=6 | 6 |
### RI NEX Measurement File

<table>
<thead>
<tr>
<th>Measurement time (receive time tags)</th>
<th>Number of tracked satellites</th>
<th>Epoch flag 0: OK</th>
<th>One satellite per row</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>97 1 9 0 7 30.00000000</td>
<td>22127685.105</td>
<td>22127685.4014</td>
<td>1</td>
</tr>
<tr>
<td>22672158.746</td>
<td>-1162184.951</td>
<td>22731130.009</td>
<td>5</td>
</tr>
<tr>
<td>22731128.396</td>
<td>-965464.169</td>
<td>22731130.009</td>
<td>5</td>
</tr>
<tr>
<td>24610920.702</td>
<td>-720085.070</td>
<td>22731130.009</td>
<td>5</td>
</tr>
<tr>
<td>20718</td>
<td>-14498133</td>
<td>22731130.009</td>
<td>5</td>
</tr>
<tr>
<td>20842</td>
<td>-14870090</td>
<td>22731130.009</td>
<td>5</td>
</tr>
</tbody>
</table>

#### Observation Data
- **G (GPS)**
- **RINEX Version/Type**
- **Comment**
- **Data Collection**
- **Measurement Time**
- **Number of Tracked Satellites**
- **Epoch Flag**
- **One Satellite per Row**

#### Example of Measurement Time
- **16**
- **97 1 9 0 7 30.00000000**

#### Example of Number of Tracked Satellites
- **7**
- **22127685.105**
- **-1162184.951**
- **-965464.169**
- **-720085.070**
- **-14498133**
- **-14870090**
### RINEX measurement file

<table>
<thead>
<tr>
<th>OBSERVATION DATA</th>
<th>G (GPS)</th>
<th>RINEX VERSION / TYPE</th>
<th>PGM / RUN BY / DATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGRINEXO V2.4.1 UX AUSLIG</td>
<td>10-JAN-97 10:19</td>
<td>COMMENT</td>
<td></td>
</tr>
<tr>
<td>Australian Regional GPS Network (ARGN) - COCOS ISLAND</td>
<td>COMMENT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FLAGs DATA COLLECTED UNDER "AS" CONDITION**

- BIT 2 OF LLI (+4)
- SNR indicators
- Synthetic P2 (A/ S=on)
- Loss of lock indicator

---

**S/N indicator**

<table>
<thead>
<tr>
<th>Time (UTC)</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997-01-01</td>
<td>9</td>
</tr>
<tr>
<td>1997-01-02</td>
<td>8</td>
</tr>
</tbody>
</table>

**Loss of lock indicator**

<table>
<thead>
<tr>
<th>Time (UTC)</th>
<th>Loss of Lock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997-01-01</td>
<td>1</td>
</tr>
<tr>
<td>1997-01-02</td>
<td>25</td>
</tr>
</tbody>
</table>

---

Master of Science in GNSS

---

J.M. Juan
\[ P = c \Delta T = c \left[ t_{rec}(T) - t_{sat}(T - \Delta T) \right] \]

\[ P_{sat}^{rec} = \rho_{rec}^{sat} + c \cdot (d_{t_{rec}} - d_{t_{sat}}) + \sum \delta \]

\[ \sum \delta = Trop_{rec}^{sat} + Ion_{rec}^{sat} + K_{rec} + K_{sat}^{sat} + \varepsilon \]

- Geometric range
- Clock offsets
- Ionospheric delay
- Tropospheric delay
- Instrumental delays
- Noise
The diagram illustrates the process of satellite-based positioning, focusing on the pseudorange calculation. The geometric range, labeled as $\rho$, is approximately 20,000 km. Various corrections and delays are highlighted, including:

- Ionospheric delay: 2 - 30 m
- Tropospheric delay: 2 - 30 m
- Receiver clock offset: <300 km
- Satellite clock offset: up to hundreds of km
- Relativistic clock correction: <13 m
- Satellite instrumental delay: ~m

The diagram also includes a box labeled $\Delta t$. The research group responsible for this content is the gAGE/UPC, a part of the research group of Astronomy and Geomatics.
Exercise:

a) Using the file coco0090.97o, generate the “txt” file coco0090.97.txt (with data ordered in columns).

b) Plot code and phase measurements for satellite PRN28 and discuss the results.

Resolution:

a) `gLAB_linux -input:cfg meas.cfg -input:obs coco0090.97o`

b) See next plots:
An example of program to read the RINEX: gLAB

RI NEX file → gLAB → txt file

The RI NEX file is converted to a “columnar format” to easily plot its contents and to analyze the measurements (the public domain free tool “gnuplot” is used in the book to make the plots).
The geometry “\( \rho \)” is the dominant term in the plot. The pattern in the figures is due to the variation of “\( \rho \)”.

\[
P_{1_{\text{sta}}} = \rho_{\text{sta}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + T_{\text{sat}} + I_{\text{1_{sta}}} + K_{\text{1_{sta}}} + K_{\text{1_{sat}}} + \varepsilon_{1}
\]
Similar plot for code measurements at $f_2$.

Notice that

- Ionosphere ($I_{on}$) and
- Instrumental delays ($K$) depend on frequency.

$$P_{2_{sta}}^{sat} = \rho_{sta}^{sat} + c \cdot (dt_{sta}^{sat} - dt_{sat}^{sat}) + Trop_{sta}^{sat} + I_{on_{sta}}^{sat} + K_{2_{sta}}^{sat} + K_{2_{sat}}^{sat} + \epsilon_2$$
Code measurements: C1,P1,P2

\[ P_{1\text{sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} + I_{\text{1\text{sta}}}^{\text{sat}} + K_{\text{1\text{sta}}}^{\text{sat}} + K_{\text{1}}^{\text{sat}} + \varepsilon_1 \]

\[ P_{2\text{sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} + I_{\text{2\text{sta}}}^{\text{sat}} + K_{\text{2\text{sta}}}^{\text{sat}} + K_{\text{2}}^{\text{sat}} + \varepsilon_2 \]

Phase measurements: L1,L2

\[ L_{1\text{sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} - I_{\text{1\text{sta}}}^{\text{sat}} + b_{\text{1\text{sta}}}^{\text{sat}} + b_{\text{1}}^{\text{sat}} + \lambda_1 N_1 + \lambda_1 \nu + \nu_1 \]

\[ L_{2\text{sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} - I_{\text{2\text{sta}}}^{\text{sat}} + b_{\text{2\text{sta}}}^{\text{sat}} + b_{\text{2}}^{\text{sat}} + \lambda_2 N_2 + \lambda_2 \nu + \nu_2 \]

Ionosphere delays code and advances phase measurements

Phase Ambiguities \( N_1, N_2 \) are integers

Wind Up
Carrier Phase measurements

The geometry \( \rho \) is the dominant term in the plot. The pattern in the figures is due to the variation of \( \rho \).

The curves are broken when the receiver loses lock (cycle-slip).

When a cycle-slip happens, the phase measurement \( L \) changes by an unknown integer number of cycles \( \mathcal{N} \).

\[
L_{\text{sta}}^{\text{sat}} = \rho_{\text{sta}}^{\text{sat}} + c \cdot (dt_{\text{sta}}^{\text{sat}} - dt_{\text{sat}}^{\text{sat}}) + Trop_{\text{sta}}^{\text{sat}} - Ion_{\text{sta}}^{\text{sat}} + b_{\text{sta}} + b_{\text{sat}}^{\text{sat}} + \mathcal{N} N_{\text{1}} + \mathcal{N} w + \nu_{\text{1}}
\]
When a cycle-slip happens, the phase measurement \( L \) changes by an unknown integer number of cycles \( N \).

\[
L_{\text{sat}}^{\text{sta}} = \rho_{\text{sat}}^{\text{sta}} + c \cdot (dt_{\text{sta}} - dt_{\text{sat}}) + Trop_{\text{sat}}^{\text{sta}} - Ion_{2\text{sta}}^{\text{sat}} + b_{2\text{sta}}^{b} + b_{2\text{sat}}^{b} + \lambda_{2}N_{2} + \lambda_{2}w + v_{2}
\]

The geometry \( \rho \) is the dominant term in the plot. The pattern in the figures is due to the variation of \( \rho \).

The curves are broken when the receiver loss the lock (cycle-slip).
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Linear Combinations of measurements:

- Geometry-free (or Ionospheric) combination.
- Ionosphere-Free combination.
- Wide-lane and Narrow-lane combinations.
1. Geometry-free (or ionospheric) combination

\[ P_{1} = P_{2} - P_{1} = Iono + ctt \]

\[ L_{1} = L_{1} - L_{2} = Iono + ctt + Ambig \]

**Code measurements:** \( C_{1}, P_{1}, P_{2} \)

\[
P_{1}^{sat}_{sta} = \rho_{sta}^{sat} + c \cdot (d_{sta} - d_{sat}^{sta}) + Trop_{sta}^{sat} + Iono_{1}^{sat}_{sta} + K_{1}^{sta} + K_{1}^{sat} + \varepsilon_{1}
\]

\[
P_{2}^{sat}_{sta} = \rho_{sta}^{sat} + c \cdot (d_{sta} - d_{sat}^{sta}) + Trop_{sta}^{sat} + Iono_{2}^{sat}_{sta} + K_{2}^{sta} + K_{2}^{sat} + \varepsilon_{2}
\]

**Carrier measurements:** \( L_{1}, L_{2} \)

\[
L_{1}^{sat}_{sta} = \rho_{sta}^{sat} + c \cdot (d_{sta} - d_{sat}^{sta}) + Trop_{sta}^{sat} - Iono_{1}^{sat}_{sta} + b_{1}^{sta} + b_{1}^{sat} + \lambda_{1}N_{1} + \lambda_{1}w + v_{1}
\]

\[
L_{2}^{sat}_{sta} = \rho_{sta}^{sat} + c \cdot (d_{sta} - d_{sat}^{sta}) + Trop_{sta}^{sat} - Iono_{2}^{sat}_{sta} + b_{2}^{sta} + b_{2}^{sat} + \lambda_{2}N_{2} + \lambda_{2}w + v_{2}
\]
1. Geometry-free (or ionospheric) combination

\[ P_i = P_2 - P_1 = \text{Iono} + \text{ctt} \]

\[ L_i = L_1 - L_2 = \text{Iono} + \text{ctt} + \text{Ambig} \]

- The pattern corresponds to the ionospheric refraction (Iono), because the other terms (K) are constant.
- Notice that code measurements are noisier.
Ionospheric effects

The ionospheric refraction depends on:
- Geographic location
- Time of day
- Time with respect to solar cycle (11y)
Ionospheric effects

The ionospheric delay ($Ion$) is proportional to the electron density integrated along the ray path ($STEC$)

$$Ion = \frac{40.3}{f^2} STEC$$

$$STEC = \int N_e(\vec{r}, t) \, dr$$
2. Ionosphere-free Combination ($P_c, L_c$)

The ionospheric refraction depends on the inverse of the squared frequency and can be removed up to 99.9% combining $f_1$ and $f_2$ signals:

\[ P_c = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2} \]

\[ L_c = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2} \]

Note: $K_{sat}$ cancels in $P_c$ and $K_{sta}$ included in $dt_{sta}$

\[ P_{c, sat}^{\text{sat}} = \rho_{sta}^{\text{sat}} + c \cdot (dt_{sta}^{\text{sat}} - dt_{sat}^{\text{sat}}) + Trop_{sta}^{\text{sat}} + \varepsilon_c \]

\[ L_{c, sat}^{\text{sat}} = \rho_{sta}^{\text{sat}} + c \cdot (dt_{sta}^{\text{sat}} - dt_{sat}^{\text{sat}}) + Trop_{sta}^{\text{sat}} + \lambda_N w_{sta}^{\text{sat}} + b_{c, sta}^{\text{sat}} + b_c^{\text{sat}} + \lambda_N (N_{1, sta}^{\text{sat}} - \frac{\lambda_W}{\lambda_2} N_{W, sta}^{\text{sat}}) + \nu_c \]

• The ionospheric refraction has been removed in $L_c$ and $P_c$

\[ \lambda_N = 10.7 \text{ cm}, \quad \lambda_W = 86.2 \text{ cm} \]

\[ N_W = N_1 - N_2 \]
Two-frequency receivers are needed to apply the ionosphere-free combination.

If a one-frequency receiver is used, a ionospheric model must be applied to remove the ionospheric refraction. The GPS navigation message provides the parameters of the Klobuchar model which accounts for more than 50% (RMS) of the ionospheric delay.
The wide-lane combination $L_W$ provides a signal with a large wavelength ($\lambda_W = 86.2 \text{ cm} \sim 4*\lambda_1$). This makes it very useful for detecting cycle-slips through the **Melbourne-Wübbena** combination: $L_W - P_N$

$$P_N = \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2}$$

$$L_W = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2}$$

**The same sign**

$$P_{N sat} = \rho_{sat}^{sta} + c \cdot (dt_{sta} - dt_{sat}) + Trop_{sat}^{sta} + Ion_{w sat}^{sta} + K_{w sat}^{sta} + K_{w sat}^{sta} + \varepsilon_N$$

$$L_{W sat} = \rho_{sat}^{sta} + c \cdot (dt_{sta} - dt_{sat}) + Trop_{sat}^{sta} + Ion_{w sat}^{sta} + b_{w sat}^{sta} + b_{w sat}^{sta} + \lambda_{w sat}^{sta} + N_{w sat}^{sta} + \nu_w$$

**The ambiguities $N_w$ are INTEGER Numbers!**

$$N_w = N_1 - N_2$$

**No wind-up**
Exercises:

1) Consider the wide-lane combination of carrier phase measurements

\[ L_w = \frac{f_1 L_1 - f_2 L_2}{f_1 - f_2} \]

where \( L_w \) is given in length units (i.e. \( L_i = \lambda_i \phi_i \)).

Show that the corresponding wavelength is:

\[ \lambda_w = \frac{c}{f_1 - f_2} \]

**Hint:**

\[ L_w = \lambda_w \phi_w \quad \phi_w = \phi_1 - \phi_2 \]

2) Assuming \( L_1, L_2 \) uncorrelated measurements with equal noise \( \sigma_L \), show that:

\[ \sigma_{L_w} = \frac{\sqrt{\gamma_{12} + 1}}{\sqrt{\gamma_{12} - 1}} \sigma_L \;
\gamma_{12} = \left( \frac{f_1}{f_2} \right)^2 \]
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Detecting cycle-slips

There is a cycle-slip of only one cycle (~20cm) ➔ How to detect it?

This cycle-slip involves millions of cycles ➔ it is easy to detect!!
Exercise:

a) Using the file 95oct18casa___r0.rnx, generate the “txt” file 95oct18casa.a (with data ordered in columns).

b) Insert a **cycle-slip** of “one wavelength” (19cm) in L1 measurement at t=5000 s (and no cycle-slip in L2).

c) Plot the measurements “L1, L1-P1, LC-PC, Lw-PN and L1-L2” and discuss which combination/s should be used to detect the cycle-slip.

Resolution:

a) gLAB_linux -input:cfg meas.cfg -input:obs 95oct18casa_r0.rnx

b) cat 95oct18casa.a | gawk '{if ($4==18) print $3,$5,$6,$7,$8}' > s18.org
cat s18.org | gawk '{if ($1>=5000) $2=$2+0.19; printf "%s %f %f %f %f
", $1,$2,$3,$4,$5}' > s18.cl

c) See next plots:
The geometry “ρ” is the dominant term in the plot. The variation of “ρ” in 1 sec may be hundreds of meters, many times greater than the cycle-slip (19 cm) ➞ the variation of ρ shadows the cycle-slip!

\[ L_{1_{sat,sta}} = ρ_{sat,sta} + c \cdot (dt_{sta} - dt_{sat}) + Trop_{sta}^{sat} - Ion_{1_{sta}}^{sat,sta} + b_{1_{sta}} + b_{sat} + λ_1 N_{1_{sta}}^{sat} + λ_1 W_{sta}^{sat} + v_1 \]
The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The $P_1$ code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.
The geometry and clock offsets have been removed. The trend is due to the Ionosphere. The \( P1 \) code noise shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of \( \lambda = 19 \text{ cm} \) (one cycle in \( L1 \)) has been introduced in \( L1 \) at \( t = 5000s \).
The geometry, clock offsets and iono have been removed. There is a constant pattern plus noise. The $P_c$ code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of $\lambda=19 \text{ cm (one cycle in L1)}$ has been introduced in L1 at $t=5000s$.

$$Lc_{\text{sta}}^{\text{sat}} - Pc_{\text{sta}}^{\text{sat}} = ctt + \text{ambig} + \epsilon$$
The geometry, clock offsets and iono have been removed. There is a constant pattern plus noise. The $P_s$ code noise also shadows the cycle-slip, and without the reference (in blue), the time where the cycle-slip happens could not be identified.

A jump of $\lambda=19\text{ cm (one cycle in L1)}$ has been introduced in L1 at $t=5000s$.

The equation is:

$$L_{I_{sta}}^{sat} - P_{I_{sta}}^{sat} = ctt + \text{ambig} + \epsilon$$

$$P_{I_{sta}}^{sat} = I_{on} + K_{I_{sta}} + K_{I_{sat}} + \epsilon_I$$

$$L_{I_{sta}}^{sat} = I_{on_I} + b_{I_{sta}} + b_{I_{sat}} + \lambda_1 N_{1_{sta}} - \lambda_2 N_{2_{sta}} + (\lambda_1 - \lambda_2) w_{sta} + v_I$$
The geometry, clock offsets and iono have been removed. There is a constant pattern plus noise. The $P_N$ code noise is under one cycle of $L_W$. Thence, the cycle-slip is clearly detected.

A jump of $\lambda = 19 \text{ cm (one cycle in L1)}$ has been introduced in $L1$ at $t=5000s$.

$$L_{W_{sta}} - P_{N_{sta}} = ctt + \text{ambig} + \varepsilon$$
The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

A jump of $\lambda = 19$ cm (one cycle in L1) has been introduced in L1 at $t=5000s$.

The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.

The geometry and clock offsets have been removed. The trend is due to the Iono. The $L_i$ carrier noise is few mm, and the variation of the ionosphere in 1 second is lower than $\lambda_1 = 19$ cm. Thence, the cycle-slip is detected.
Summary

Session 3b, exercise 2b: Cycle-slip detection with L1-P1, PRN 18

Session 3b, exercise 2c: Cycle-slip detection with LC-PC, PRN 18

Master of Science in GNSS

@ J. Sanz & J.M. Juan
The cycle-slips are detected by the Ionospheric combination (LI=L1-L2) and the Melbourne Wübbena (MW=Lw-PN).

Two independent combinations, LI and Lw, allow to detect two independent cycle-slips (in L1 and L2 phase measur.).

Notice that, from L1, L2 is not possible to detect small cycle-slips.
## Summary of Cycle-slip detectors

<table>
<thead>
<tr>
<th>COMBINATION</th>
<th>MEAS</th>
<th>Combination Noise ($\sigma$)</th>
<th>$\lambda$</th>
<th>$\sigma/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$-$P_1$</td>
<td>$-2 \cdot I_{on} + K + \lambda_1 \cdot N_1$</td>
<td>$\sigma_{L_1-P_1} \approx \sigma_{P_1} = 30 \text{ cm}$</td>
<td>$\lambda_1 = 19.0 \text{ cm}$</td>
<td>1.58</td>
</tr>
<tr>
<td>$L_c$-$P_c$</td>
<td>$k_c + \lambda_N \cdot R_c$</td>
<td>$\sigma_{L_C-P_C} \approx \sigma_{P_C} = 2.98 \sigma_P = 89 \text{ cm}$</td>
<td>$\lambda_N = 10.7 \text{ cm}$</td>
<td>8.32</td>
</tr>
<tr>
<td>$L_I$-$P_I$</td>
<td>$k_I + \lambda_1 \cdot N_1 - \lambda_2 \cdot N_2$</td>
<td>$\sigma_{L_I-P_I} \approx \sqrt{2} \sigma_P = 42 \text{ cm}$</td>
<td>$\lambda_2 - \lambda_1 = 5.4 \text{ cm}$</td>
<td>7.78</td>
</tr>
<tr>
<td>$L_W$-$P_N$</td>
<td>$\lambda_W \cdot N_w$</td>
<td>$\sigma_{L_W-P_N} \approx \sigma_{P_N} = \sigma_P / \sqrt{2} = 21 \text{ cm}$</td>
<td>$\lambda_W = 86.2 \text{ cm}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$L_I$</td>
<td>$I_{on} + k_I + \lambda_1 \cdot N_1 - \lambda_2 \cdot N_2$</td>
<td>$\sigma_{L_I} = \sqrt{2} \sigma_{L_1} = 3 \text{ mm}$</td>
<td>$\lambda_2 - \lambda_1 = 5.4 \text{ cm}$</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
   3.1 Cycle-slip Detection Algorithms
4. Carrier smoothing of code pseudorange.
Cycle-slip detector based on carrier phase data: The Geometry-free combination

**Input data:** Geometry-free combination of carrier phase measurements

\[
L_I = L_1(s; k) - L_2(s; k)
\]

**Output:** [satellite, time, cycle-slip flag].

For each epoch \((k)\)

For each tracked satellite \((s)\)

- Declare cycle slip when data gap greater than \(tol_{\Delta t}\).\(^{16}\)
- Fit a second-degree polynomial \(p(s; k)\) to the previous values (after the last cycle-slip) \([L_I(s; k - N_I), \ldots, L_I(s; k - 1)]\).
- Compare the measured \(L_I(s; k)\) and the predicted value \(p(s; k)\) at epoch \(k\). If the discrepancy exceeds a given threshold, then declare cycle slip. That is,

\[
\text{if } |L_I(s; k) - p(s; k)| > \text{threshold} \text{ then cycle slip.}
\]
- Reset algorithm after cycle slip.

**End**

End
Under not disturbed ionospheric conditions, the geometry-free combination performs as a very precise and smooth test signal, driven by the ionospheric refraction. Although, for instance, the jump produced by a simultaneous one-cycle slip in both signals is smaller in this combination than in the original signals ($\lambda_2 - \lambda_1 = 5.4\text{cm}$), it can provide reliable detection even for small jumps.
Cycle-slip detector based on code and carrier phase data: The Melbourne-Wübbena combination

**Input data:** Melbourne-Wübbena combination

**Output:** [satellite (PRN), time, cycle-slip flag]

For each epoch ($k$):

For each tracked satellite ($s$):

- Declare cycle-slip when data hole greater than $tol_{\Delta t}$ (e.g., 60 s).
- If no data hole larger than $tol_{\Delta t}$, thence:
  - Compare the measurement $B_W(s; k)$ at the epoch $k$ with the mean bias $m_{BW}(s; k - 1)$ computed from the previous values.
  - If the discrepancy is over a threshold $= K_{factor} \cdot S_{BW}$ (e.g., $K_{factor} = 4$), declare cycle-slip. That is:
    \[
    \text{If } |B_W(s; k) - m_{BW}(s; k - 1)| > K_{factor} \cdot S_{BW}(s; k - 1),
    \]
    Thence, cycle-slip.
- Update the mean and sigma values according to the equations:
  \[
  m_{BW}(s; k) = \frac{k - 1}{k} m_{BW}(s; k - 1) + \frac{1}{k} B_W(s; k)
  \]
  \[
  S_{BW}^2(s; k) = \frac{k - 1}{k} S_{BW}^2(s; k - 1) + \frac{1}{k} (B_W(s; k) - m_{BW}(s; k - 1))^2
  \]
  \[\text{(4.24)}\]

Note the $S_{BW}$ is initialised with an a priori $S_0 = \lambda_w / 2$.

---

J.M. Juan
The Melbourne-Wübbena combination has a double benefit:

- The enlargement of the ambiguity spacing, thanks to the larger wavelength $\lambda_W = 86.2\text{cm}$.
- The noise is reduced by the narrow-lane combination of code measurement.

Nevertheless, in spite of these benefits, the performance is worse than in the previous carrier-phase-only based detector and it is used as a secondary test.
Exercises:

1) Show that $\Delta N_1 = 9$ and $\Delta N_2 = 7$
produces jumps of few millimetres in the geometry-free combination.

2) Show that no jump happens in the geometry-free combination when
$\Delta N_1 / \Delta N_2 = 77 / 60$. In particular when $\Delta N_1 = 77$ and $\Delta N_2 = 60$ the
jump in the wide-lane combination is: $17\lambda_w = 15m$

Hint: Consider the following relationships (from [RD-1]):

The effect of a jump in the integer ambiguities in terms of $\Delta N_1$, $\Delta N_2$
and $N_W$ is given next:

\[
\begin{align*}
\Delta \Phi_w, \Delta \Phi_r, \Delta \Phi_c \text{ variations} \\
\Delta \Phi_w &= \lambda_w \Delta N_W = \lambda_w (\Delta N_1 - \Delta N_2) \\
\Delta \Phi_r &= \lambda_1 \Delta N_1 - \lambda_2 \Delta N_2 = (\lambda_2 - \lambda_1) \Delta N_1 + \lambda_2 \Delta N_W \\
\Delta \Phi_c &= \lambda_N \left( \frac{\lambda_w}{\lambda_1} \Delta N_1 - \frac{\lambda_w}{\lambda_2} \Delta N_2 \right) = \lambda_N \left( \Delta N_1 + \frac{\lambda_w}{\lambda_2} \Delta N_W \right)
\end{align*}
\]
Example of Single frequency Cycle-slip detector

Input data: Code pseudorange \( (P_1) \) and carrier phase \( (L_1) \) measurements.

Output: [satellite (PRN), time, cycle-slip flag]

For each epoch \( (k) \)

For each tracked satellite \( (s) \)

- Declare cycle-slip when data hole greater than \( tol_{\Delta t} \)\(^{24}\).
- If no data hole larger than \( tol_{\Delta t} \), thence:
- Update an array with the last \( N \) differences of

\[
d(s; k) = L_1(s; k) - P_1(s; k)
\]

That is: \([d(s; k-N), \ldots, d(s; k-1)]\)

- Compute the mean and sigma discrepancy over the previous \( N \) samples \([k-N, \ldots, k-1]\):

\[
m_d(s; k-1) = \frac{1}{N} \sum_{i=1}^{N} d(s; k-i)
\]

\[
m_{d2}(s; k-1) = \frac{1}{N} \sum_{i=1}^{N} d^2(s; k-i)
\]

\[
S_d(s; k-1) = \sqrt{m_{d2}(s; k-1) - m_d^2(s; k-1)}
\]

- Compare the difference at the epoch \( k \) with the mean value of differences computed over the previous \( N \) samples window. If the value is over a threshold \( n_T \times S_d \) (e.g., \( n_T = 5 \)), declare cycle-slip\(^{25}\).

That is:

\[
If \ |d(s; k) - m_d(s; k-1)| > n_T S_d(s; k-1),
\]

Thence, cycle-slip.

End
This detector is affected by the code pseudorange noise and multipath as well as the divergence of the ionosphere.

Higher sampling rate improves detection performance, but shortest jumps can still escape from this detector.

On the other hand, a minimum number of samples is needed for filter initialization in order to ensure a reliable value of sigma for the detection threshold.

More details, exercises and examples of software code implementation of these detectors can be found in [RD-1] and [RD-2].
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Carrier smoothing of code pseudorange

The noisy (but unambiguous) code pseudorange can be smoothed with the precise (but ambiguous) carrier. A simple algorithm is given next:

**Hatch filter:**

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right)
\]

where \( \hat{P}(1) = P(1) \) and

- \( n = k; \quad k < N \)
- \( n = N; \quad k \geq N \)

This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \left\langle P - L \right\rangle_{(k)}
\]
This algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement:

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \langle P - L \rangle_{(k)}
\]
Hatch filter: Carrier-smoothed code. N=100 epochs

\[
\hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \langle P - L \rangle_{(k)}
\]
Hatch filter: Carrier-smoothed code. N=100 epochs

\[ \hat{P}(k) = \frac{1}{n} P(k) + \frac{n-1}{n} \left( \hat{P}(k-1) + L(k) - L(k-1) \right) = L(k) + \langle P - L \rangle_{(k)} \]
Code-carrier divergence: SF smoother

Time varying ionosphere induces a bias in the single frequency (SF) smoothed code when it is averaged in the smoothing filter (Hatch filter).

Let:

\[ P_1 = \rho + I_1 + \varepsilon_1 \]
\[ L_1 = \rho - I_1 + B_1 + \zeta_1 \]

where \( \rho \) includes all non dispersive terms (geometric range, clock offsets, troposphere) and \( I_1 \) represents the frequency dependent terms (ionosphere and DCBs). \( B_1 \) is the carrier ambiguity, which is constant along continuous carrier phase arcs and \( \varepsilon_1, \zeta_1 \) account for code and carrier multipath and thermal noise.

thence,

\[ P_1 - L_1 = 2I_1 - B_1 + \varepsilon_1 \quad \Rightarrow 2I_1 : \text{Code-carrier divergence} \]

Substituting \( P_1 - L_1 \) in Hatch filter equation

\[ \hat{P}(k) = L(k) + \langle P - L \rangle_{(k)} = \rho(k) - I_1(k) + B_1 + \langle 2I_1 - B_1 \rangle_{(k)} = \]

\[ = \rho(k) + I_1(k) + 2\left( \langle I_1 \rangle_{(k)} - I_1(k) \right) \]

\[ \Rightarrow \hat{P}_1 = \rho + I_1 + \text{bias}_1 + \nu_1 \]

where, being the ambiguity term \( B_1 \) a constant bias, thence \( \langle B_1 \rangle_{(k)} \approx B_1 \), and cancels in the previous expression.

where \( \nu_1 \) is the noise term after smoothing.
PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 DFree smoothed (3600)

N=3600 s

STEC PRN03 (shifted)

1.546*(L1-L2)
PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 DFree smoothed (3600)

meters

time (s)
Halloween storm

Data File: amc23030.03o_1Hz

\[ N=100 \] (i.e. filter smoothing time constant \( \tau=100 \text{ sec} \)).
Halloween storm

Carrier-smoothed pseudorange: DFree

Divergence-Free (Dfree) smoother:

With two frequency carrier measurements a combination of carriers with the same ionospheric delay (the same sign) as the code can be generated:

\[ L_{1,DF} = L_1 + 2\tilde{\alpha}_1(L_1 - L_2) = \rho + I_1 + B_{1,DF} + \zeta_{1,DF} \]

\[ \tilde{\alpha}_1 = \frac{f_2^2}{f_1^2 - f_2^2} = \frac{1}{\gamma - 1} = 1.545 \]

\[ \gamma = \left(\frac{77}{60}\right)^2 \]

With this new combination we have:

\[ P_1 = \rho + I_1 + \varepsilon_1 \]

\[ L_{1,DF} = \rho + I_1 + B_{1,DF} + \zeta_{1,DF} \]

Thence,

\[ P_1 - L_{1,DF} = B_{1,DF} + \varepsilon_1 \]

\[ \Rightarrow \text{No Code-carrier divergence!} \]

This smoothed code is immune to temporal gradients (unlike the SF smoother), being the same ionospheric delay as in the original raw code (i.e. \(I_1\)). Nevertheless, as it is still affected by the ionosphere, its spatial decorrelation must be taken into account in differential positioning.

\[ \hat{P}_{1,DF} = \rho + I_1 + \nu_{12} \]
PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 DFree smoothed (3600)

meters

time (s)
Carrier-smoothed pseudorange: I Free

Ionosphere-Free (I free) smoother:
Using both code and carrier dual-frequency measurements, it is possible to remove the frequency dependent effects using the ionosphere-free combination of code and carriers (PC and LC). Thence:

\[
P_C = \rho + \varepsilon_{P_C} \\
L_C = \rho + B_{LC} + \nu_{LC}
\]

\[
P_{IFree} \equiv P_C = \frac{\gamma P_1 - P_2}{\gamma - 1} ; \quad L_{IFree} \equiv L_C = \frac{\gamma L_1 - L_2}{\gamma - 1} \quad \gamma = \left(\frac{77}{60}\right)^2
\]

\[
P_C - L_C = B_C + \varepsilon_{P_C} \implies \hat{P}_{IFree} \equiv \hat{P}_C = \rho + \nu_{IFree}
\]

\[
\sigma_{P_C} = \sqrt{\frac{\gamma^2 + 1}{\gamma - 1}} \sigma_{P_1} \div 3 \sigma_{P_1}
\]

This smoothed is based on the ionosphere-free combination of measurements, and therefore it is unaffected by either the spatial and temporal inospheric gradients, but has the disadvantage that the noise is amplified by a factor 3 (using the legacy GPS signals).
Vertical range: [-5 : 5]

PRN03: C1 3600s smoothing and divergence of ionosphere

Vertical range: [-15:15]

Ionosphere-Free combination smoothing: 3600 seconds
Exercise:

Justify that the ionosphere-free combination (PC) is (obviously) not affected by the code-carrier divergence, but it is 3 times noisier.
**C1, L1**

PRN03: C1 3600s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (3600)
- C1 Dfree smoothed (3600)

**N=3600**

PRN03: C1 360s smoothing and divergence of ionosphere

- C1 Raw
- C1 SF smoothed (360)
- C1 Dfree smoothed (360)

**N=360**

**PC, LC**

Ionosphere-Free combination smoothing: 3600 seconds

- Ifree raw
- Ifree smth (3600s)

**N=3600**

Ionosphere-Free combination smoothing: 360 seconds

- Ifree raw
- Ifree smth (360s)

**N=360**
Halloween storm

Data File: amc23030.03o_1Hz

\[ N = 100 \text{ (i.e. filter smoothing time constant } \tau = 100 \text{ sec)} \]
Contents

1. Review of GNSS measurements.
2. Linear combinations of measurements.
3. Carrier cycle-slips detection.
4. Carrier smoothing of code pseudorange.
Multipath

One or more reflected signals reach the antenna in addition to the direct signal. Reflective objects can be earth surface (ground and water), buildings, trees, hills, etc.

It affects both code and carrier phase measurements, and it is more important at low elevation angles.

Code: up to 1.5 chip-length $\Rightarrow$ up to 450m for C1 [theoretically]
Typically: less than 2-3 m.

Phase: up to $\lambda/4$ $\Rightarrow$ up to 5 cm for L1 and L2 [theoretically]
Typically: less than 1 cm
**Exercise**

Plot code and phase geometry-free combination for satellite PRN 15 of file 97jan09coco_r0.rnx and discuss the results.

*Butterfly shape:*

High multipath for low elevation rays (when satellite rises and sets)
\[ M_{Pc} = P_c - L_c \]
PC code multipath: upc3: PRN20

\[ M_{Pc} = Pc - Lc \]
\[ M_{MW} = P_N - L_W \]
\[ M_{MW} = P_N - L_W \]
After one year, the directions of the Sun and Aries coincide again, but the number of laps relative to the Sun (solar days) is one less than those relative to Aries (sidereal days).

\[
\frac{24h}{365.2422} = 3^m 56^s
\]

Thus, a **sidereal day is shorter** than a solar day for about \(3^m 56^s\)
Receiver noise and multipath

Barcelona, Spain: 2005 5 29
Receiver: NovAtel OEM3, Antenna: NovAtel 600 (Pinwheel)

GPS standalone (C1 code)

10,000 €
Receiver noise and multipath

GPS standalone (C1 code)

Same environment!

Barcelona, Spain: 2005 5 29
Receiver: Trimble Lassen SK-II, Antenna: Compact Magnetic-Mount

North (meters)

East (meters)

100 €
References


Thank you!