Lecture 6
Differential positioning with Code pseudoranges

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5 March 2017
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Error mitigation: DGNSS residual error

Errors are similar for users separated tens, even hundred of kilometres, and these errors vary ‘slowly’ with time. That is, the errors are correlated on space and time.

The spatial decorrelation depends on the error component (e.g. Clocks not decorrelate, ionosphere ~100km...). Thence, long baselines need a reference stations network.
Linear model for Differential Positioning

Code and carrier measurements

\[ P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + M_i^j + \nu_{p_i}^j \]
\[ L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda \omega_i^j + \lambda N_i^j + b_i + b^j + m_i^j + \nu_{\lambda_i}^j \]

When approximate values of both receiver and satellite APC positions are taken, a linearization around them yields:

\[ \rho_i^j = \rho_{0i}^j - \hat{\rho}_{0i}^j \cdot \Delta r_i + \hat{\rho}_{0i}^j \cdot \Delta r_i^j \]

\[ \hat{\rho}_{0i}^j = \frac{r_{0i}^j - r_{oi}}{\|r_{0i}^j - r_{oi}\|} \]

\[ \Delta r_i : \text{Receiver coordinates error} \]
\[ \Delta r_i^j : \text{Satellite coordinates error} \]
Linear model for Differential Positioning

Code and carrier measurements

\[
P_i^j = \rho_i^j + c(\Delta t_i - \Delta t^j) + T_i^j + I_i^j + K_i + K^j + M_i^j + \nu_{p_i}^j
\]

\[
L_i^j = \rho_i^j + c(\Delta t_i - \Delta t^j) + T_i^j - I_i^j + \lambda \omega_i^j + \lambda N_i^j + b_i + b^j + m_i^j + \nu_{L_i}^j
\]

**Single difference**

\[
(\bullet)^j_{ru} \equiv \Delta(\bullet)^j_{ru} = (\bullet)^j_u - (\bullet)^j_r
\]

\[
P_u^j = \rho_u^j + c(\Delta t_u - \Delta t^j) + T_u^j + I_u^j + K_u + K^j + \nu_{p_u}^j
\]

\[
P_r^j = \rho_r^j + c(\Delta t_r - \Delta t^j) + T_r^j + I_r^j + K_r + K^j + \nu_{p_r}^j
\]

\[
P_{ru}^j = \rho_{ru}^j + c \Delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \nu_{p_{ru}}^j
\]

The same for the carrier :

\[
L_{ru}^j = \rho_{ru}^j + c \Delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + b^j + m_{ru}^j + \nu_{L_{ru}}^j
\]
Linear model for Differential Positioning

Code and carrier measurements

\[ P_i^j = \rho_i^j + c(\delta t_i - \delta t^j) + T_i^j + I_i^j + K^j + M_i^j + \nu_{p_i}^j \]

\[ L_i^j = \rho_i^j + c(\delta t_i - \delta t^j) + T_i^j - I_i^j + \lambda \omega_i^j + \lambda N_i^j + b_i + b^j + m_i^j + \nu_{\lambda_i}^j \]

**Single difference**

\[ (\bullet)^{j}_{ru} \equiv \Delta(\bullet)^{j}_{ru} = (\bullet)^{j}_u - (\bullet)^{j}_r \]

\[ P_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \nu_{p_{ru}}^j \]

\[ L_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + \nu_{\lambda_{ru}}^j \]

**Single difference cancels:**

- Satellite clock \((\delta t^j)\)
- Satellite code instrumental delays \((K^j)\)
- Satellite carrier instrumental delays \((b^j)\)

**Single differences mitigate/remove errors due**

- Satellite Ephemeris \((\Delta r^j)\)
- Ionosphere \((I_i^j)\)
- Troposphere \((T_i^j)\)
- Wind-up \((\omega_i^j)\)

**The residual errors will depend upon the baseline length.**
Single-Difference of measurements (corrected by geometric range!!)

\[ \Delta(L_1 - \rho) \equiv L_{1ru}^{sat} - \rho_{ru}^{sat} \]

\[ \Delta(P_1 - \rho) \equiv P_{1ru}^{sat} - \rho_{ru}^{sat} \]

Dif. Wind-up: Very small

Dif. Receiver clock:
Main variations Common for all satellites

Dif. Tropo. and Iono.:
Small variations

Dif. Instrumental delays and carrier ambiguities: constant
Single-Difference of measurements (corrected by geometric range!!)

\[ \Delta(L_1 - \rho) = L_{sat}^{ru} - \rho_{ru}^{sat} \]

\[ \Delta(P_1 - \rho) = P_{sat}^{ru} - \rho_{ru}^{sat} \]
Single-Difference of measurements (corrected by geometric range!!)

\[ \Delta(L_1 - \rho) \equiv L_{1ru} - \rho_{ru}^{sat} \]

\[ \Delta(P_1 - \rho) \equiv P_{1ru} - \rho_{ru}^{sat} \]

Dif. Wind-up: Very small

Dif. Receiver clock:
Main variations Common
for all satellites

Dif. Tropo. and Iono.:
Small variations

Dif. Instrumental delays and carrier
ambiguities: constant
**Linear model for Differential Positioning**

**Code and carrier measurements**

\[ P_i^j = \rho_i^j + c(\delta t_i - \delta t^j) + T_i^j + I_i^j + K_i + K^j + \nu_{p_i}^j \]

\[ L_i^j = \rho_i^j + c(\delta t_i - \delta t^j) + T_i^j - I_i^j + \lambda \omega_i^j + \lambda N_i^j + b_i + b^j + \nu_{L_i}^j \]

where:

\[ \rho_i^j = \rho_{0i}^j - \hat{\rho}_{0i}^j \cdot \Delta r_i + \hat{\rho}_{0i}^j \cdot \Delta r^j \]

**Single difference**

\[ (\bullet)^j_{ru} \equiv \Delta(\bullet)^j_{ru} = (\bullet)^j_u - (\bullet)^j_r \]

\[ P^j_{ru} = \rho^j_{ru} + c \delta t_{ru} + T^j_{ru} + I^j_{ru} + K_{ru} + \epsilon^j_{ru} \]

\[ L^j_{ru} = \rho^j_{ru} + c \delta t_{ru} + T^j_{ru} - I^j_{ru} + \lambda \omega^j_{ru} + \lambda N^j_{ru} + b_{ru} + \nu^j_{L_{ru}} \]

where:

\[ \rho^j_{ru} = \rho^j_u - \rho^j_r \]

With some algebraic manipulation, it follows:

\[ \rho^j_{ru} = \rho^j_{0ru} - \hat{\rho}_{0u}^j \cdot \Delta r_{ru} - \hat{\rho}_{0ru}^j \cdot \epsilon_{site} + \hat{\rho}_{0ru}^j \cdot \epsilon_{eph} \]

being:

\[ \rho^j_{0ru} \equiv \rho^j_{0u} - \rho^j_{0r} ; \quad \Delta r^j_{ru} \equiv \Delta r^j_u - \epsilon_{site} \]

Finally:

\[ r_u = r_{0u} + \Delta r^j_{ru} + \epsilon_{site} \]
Linear model for Differential Positioning

and where the $\Delta r_{ru}$ estimate will be affected in turn by the ephemeris and sitting site errors as:

$$
\rho_{ru}^j = \rho_{0ru}^j - \hat{\rho}_{0u}^j \cdot \Delta r_{ru} - \hat{\rho}_{0ru}^j \cdot \varepsilon_{site}^j + \hat{\rho}_{0ru}^j \cdot \varepsilon_{eph}^j
$$

Range error due to reference station coordinates uncertainty
Range error due to Sat. coordinates uncertainty

Thence, taking into account the relationships:

$$
\hat{\rho}_{0ru}^j \cdot \varepsilon_{site}^j \leq \frac{b}{\rho_u^j} \| \varepsilon_{site} \| ; \quad \hat{\rho}_{0ru}^j \cdot \varepsilon_{eph}^j \leq \frac{b}{\rho_u^j} \| \varepsilon_{eph} \|
$$

and being $\rho_u^j \approx 20000$km it follows that for a baseline $b = 20$km

$$
\frac{b}{\rho_u^j} \approx \frac{1}{1000} \Rightarrow
$$

The effect of 5 metres error in orbits or in site coordinates is less than 5mm in range for the estimation of $\Delta r_{ru}$.

But, the user position estimate will be shifted by the error in the site coordinates $r_u = r_{0u} + \Delta r_{ru} + \varepsilon_{site}$
Exercise:

Let be: \( \rho^j_{ru} = \rho^j_u - \rho^j_r \)

where \( \rho^j_i = \rho^j_{0i} - \hat{\rho}^j_{0i} \cdot \Delta r_i + \hat{\rho}^j_{0i} \cdot \Delta r^j \) (from Taylor expansion)

Show that the Single Differences are given by:

\[
\rho^j_{ru} = \rho^j_{0ru} - \hat{\rho}^j_{0u} \cdot \Delta r_{ru} - \hat{\rho}^j_{0ru} \cdot \Delta r_r + \hat{\rho}^j_{0ru} \cdot \Delta r^j
\]

being: \( \rho^j_{0ru} \equiv \rho^j_{0u} - \rho^j_{0r} \); \( \Delta r_{ru} \equiv \Delta r_u - \Delta r_r \)
Hint: Linear model for Differential Positioning

Code and carrier measurements

\[ P_i^j = \rho_i^j + c(\delta t_i - \delta t^j) + T_i^j + I_i^j + K + K^j + \nu_{\rho_i}^j \]

\[ L_i^j = \rho_i^j + c(\delta t_i - \delta t^j) + T_i^j - I_i^j + \lambda \omega_i^j + \lambda N_i^j + b_i + b^j + \nu_{\lambda_i}^j \]

where:

\[ \rho_i^j = \rho_{0i}^j - \tilde{\rho}_{0i}^j \cdot \Delta r_i + \tilde{\rho}_{0i}^j \cdot \Delta r^j \]

Single difference

\[ (\bullet)_u^j \equiv \Delta (\bullet)_r^j = (\bullet)_u^j - (\bullet)_r^j \]

\[ P_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \varepsilon_{ru}^j \]

\[ L_{ru}^j = \rho_{ru}^j + c \delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda \omega_{ru}^j + \lambda N_{ru}^j + b_{ru} + \nu_{\lambda_{ru}}^j \]

where:

\[ \rho_{ru}^j = \rho_u^j - \rho_r^j \]

\[ \rho_{ru}^j = \rho_{0ru}^j - \tilde{\rho}_{0u}^j \cdot \Delta r_u + \tilde{\rho}_{0r}^j \cdot \Delta r_r + \tilde{\rho}_{0r}^j \cdot \Delta r^j - \tilde{\rho}_{0r}^j \cdot \Delta r^j \]

\[ = \rho_{0ru}^j - \tilde{\rho}_{0u}^j \cdot \Delta r_{ru} - \tilde{\rho}_{0r}^j \cdot \Delta r_r + \tilde{\rho}_{0r}^j \cdot \Delta r^j \]

being:

\[ \rho_{0ru}^j \equiv \rho_{0u}^j - \rho_{0r}^j \quad \Delta r_{ru} \equiv \Delta r_u - \Delta r_r \]
Linear model for Differential Positioning

\[
\rho_{ru}^j = \rho_{0ru}^j - \hat{\rho}_{0u}^j \cdot \Delta r_{ru} - \hat{\rho}_{0ru}^j \cdot \Delta r_{j} + \hat{\rho}_{0ru}^j \cdot \Delta r_{j} \quad \text{with} \quad \Delta r_{ru} \equiv \Delta r_{u} - \Delta r_{r}
\]

Let’s assume that:

- The satellite coordinates are known with an uncertainty \( \Delta r_{j} \equiv \epsilon_{eph}^j \) (i.e. \( r_{r} = r_{0r} + \epsilon_{site} \))
- The reference station coordinates are known with an uncertainty \( \Delta r_{r} \equiv \epsilon_{site} \)

Thence, the user position can be computed from the estimate, with an error \( \epsilon_{site} \), as:

\[
\mathbf{r}_{u} = r_{0u} + \Delta r_{ru} + \epsilon_{site}
\]

and where the \( \Delta r_{ru} \) estimate will be affected in turn by the ephemeris and sitting site errors as:

\[
\rho_{ru}^j = \rho_{0ru}^j - \hat{\rho}_{0u}^j \cdot \Delta r_{ru} - \hat{\rho}_{0ru}^j \cdot \epsilon_{site} + \hat{\rho}_{0ru}^j \cdot \epsilon_{eph}^j
\]

Range error due to reference station coordinates uncertainty
Range error due to Sat. coordinates uncertainty

Backup

\[
\mathbf{r}_{u} = r_{0u} + \Delta r_{u} = r_{0u} + \Delta r_{ru} + \Delta r_{r} = r_{0u} + \Delta r_{ru} + \epsilon_{site}
\]
Linear model for Differential Positioning

and where the $\Delta \mathbf{r}_{ru}$ estimate will be affected in turn by the ephemeris and sitting site errors as:

$$\rho_{ru}^j = \rho_{0ru}^j - \hat{\rho}_{0u}^j \cdot \Delta \mathbf{r}_{ru} - \hat{\rho}_{0ru}^j \cdot \mathbf{e}_{site}^j + \hat{\rho}_{0ru}^j \cdot \mathbf{e}_{eph}^j$$

Range error due to reference station coordinates uncertainty

Range error due to Sat. coordinates uncertainty

Thence, taking into account the relationships:

$$\hat{\rho}_{0ru}^j \cdot \mathbf{e}_{site}^j \leq \frac{b}{\rho_u^j} \| \mathbf{e}_{site}^j \| ; \quad \hat{\rho}_{0ru}^j \cdot \mathbf{e}_{eph}^j \leq \frac{b}{\rho_u^j} \| \mathbf{e}_{eph}^j \|$$

and being $\rho_u^j \approx 20000\text{km}$ it follows that for a baseline $b = 20\text{km}$

$$\frac{b}{\rho_u^j} \approx \frac{1}{1000} \Rightarrow$$

The effect of 5 metres error in orbits or in site coordinates is less than 5 mm in range for the estimation of $\Delta \mathbf{r}_{ru}$.

But, the user position estimate will be shifted by the error in the site coordinates $\mathbf{r}_u = \mathbf{r}_{0u} + \Delta \mathbf{r}_{ru} + \mathbf{e}_{site}$.

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Exercise:

Demonstrate the following relationship:

$$\hat{\rho}_{0ru} \cdot \varepsilon_{site} \leq \frac{b}{\rho} \| \varepsilon_{site} \|$$

Hint:

$$\hat{\rho}_{0ru} \cdot \varepsilon_{site} = (\hat{\rho}_{0u} - \hat{\rho}_{0r}) \cdot \varepsilon_{site}$$

$$\approx \left( \frac{\rho_{0u} - \rho_{0r}}{\rho} \right) \cdot \varepsilon_{site} \leq \frac{b}{\rho} \| \varepsilon_{site} \|$$

Note: the following approaches have been taken:

$$\rho_{0r} \approx \rho_{0u} \approx \rho \quad \Rightarrow$$

$$\hat{\rho}_{0r} = \frac{\rho_{0r}}{\rho_{0r}} \approx \frac{\rho_{0r}}{\rho}$$

$$\hat{\rho}_{0u} = \frac{\rho_{0u}}{\rho_{0u}} \approx \frac{\rho_{0u}}{\rho}$$

$$b = \| r_{ru} \| \approx \| r_{0ru} \| \quad u \cdot v \leq \| u \| \| v \|$$
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Geographic decorrelation of ephemeris errors

Differential range error due to satellite orbit error

\[ \delta \rho = \ddot{\rho} \cdot \frac{\rho_{\text{user}}}{\rho_{\text{ref}}} - \ddot{\rho} \cdot \frac{\rho_{\text{user}}}{\rho_{\text{ref}}} \]

A conservative bound:

\[ \delta \rho < \frac{b}{\rho} \varepsilon \]

with a baseline \( b = 20km \)

\[ \delta \rho < \frac{20}{20000} \varepsilon = \frac{1}{1000} \varepsilon \]
Satellite location error $\varepsilon$

Range error from CREU and EBRE

Differential range error from between CREU and EBRE

$\delta\rho = \frac{1}{\rho_{\text{user}}} - \frac{1}{\rho_{\text{ref}}} \cdot \varepsilon$
Range error from CREU and EBRE

\[ \delta \rho = \bar{\varepsilon} \cdot \frac{\bar{\rho}_{\text{user}}}{\rho_{\text{user}}} - \bar{\varepsilon} \cdot \frac{\bar{\rho}_{\text{ref}}}{\rho_{\text{ref}}} \]

CREU: SPP Along-Track orbit error

CREU-EBRE: 288 km: SPP 2000m Along-Track orbit error

Differential range error from between CREU and EBRE

288 km of baseline

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Orbit Errors over the hyperboloid will not produce differential range errors.

\[
\tilde{\rho}_1 = \rho_1 + \Delta \rho_{\text{eph}} \\
\tilde{\rho}_2 = \rho_2 + \Delta \rho_{\text{eph}} \\
\Rightarrow \tilde{\rho}_1 - \tilde{\rho}_2 = \rho_1 - \rho_2 = \text{constant} \Rightarrow \delta \rho = 0
\]
Orbit Errors over the hyperboloid will not produce differential range errors.

\[ \tilde{\rho}_1 = \rho_1 + \Delta \rho_{\varepsilon_{eph}} \]
\[ \tilde{\rho}_2 = \rho_2 + \Delta \rho_{\varepsilon_{eph}} \]

\[ \Rightarrow \quad \tilde{\rho}_1 - \tilde{\rho}_2 = \rho_1 - \rho_2 = \text{constant} \Rightarrow \quad \delta \rho = 0 \]
Errors over the hyperboloid (i.e. \( \rho_B - \rho_A = ctt \)) will not produce differential range errors.

The highest error is given by the vector \( \hat{\rho} \), orthogonal to the hyperboloid and over the plain containing the baseline vector \( \hat{b} \) and the LoS vector \( \hat{\rho} \).

Note:
Being the baseline \( b \) much smaller than the distance to the satellite, we can assume that the LoS vectors from A and B receives are essentially identical to \( \rho \).
That is, \( \rho_B \equiv \rho_A \equiv \rho \)

\[
\mathbf{u} = \hat{\rho} \times (\hat{b} \times \hat{\rho}) = \hat{b} (\hat{\rho}^T \cdot \hat{\rho}) - \hat{\rho} (\hat{\rho}^T \cdot \hat{b}) = \mathbf{I} \hat{b} - (\hat{\rho} \cdot \hat{\rho}^T) \hat{b} = (\mathbf{I} - \hat{\rho} \cdot \hat{\rho}^T) \hat{b}
\]

Note: \( \mathbf{u} = \sin \phi \hat{\rho} \)

Differential range error \( \delta \rho \) produced by an orbit error \( \varepsilon_\parallel \) parallel to vector \( \hat{u} \)

Let \( \delta \varepsilon \equiv \varepsilon_\parallel \)

\[
\delta \rho = \delta (\rho_B - \rho_A) = 2 \delta a = \frac{2 \partial a}{\partial \varepsilon} \partial \varepsilon = \frac{2 \partial a}{\partial \phi} \partial \phi \partial \varepsilon = -b \sin \phi \frac{\partial \phi}{\partial \varepsilon} \partial \varepsilon \\
\approx -b \sin \phi \frac{1}{\rho} \delta \varepsilon
\]

Note: \( \varepsilon_\parallel \perp \hat{\rho} \Rightarrow \delta \varepsilon \approx \rho \delta \phi \)

\[
\delta \rho = -b \sin \phi \frac{\varepsilon^T \cdot \hat{u}}{\rho} = -\varepsilon^T (\sin \phi \hat{\rho}) \frac{b}{\rho}
\]

Thence:

\[
\delta \rho = -\varepsilon^T (\mathbf{I} - \hat{\rho} \cdot \hat{\rho}^T) \frac{b}{\rho}
\]

Where: \( b = b \hat{b} \)
is the baseline vector

Tip:
To be a vector orthogonal to the LoS \( \hat{\rho} \), hence, \( \varepsilon_\parallel = \varepsilon^T \hat{\rho} \)
**ORBIT TEST**: Broadcast orbits Along-track Error (PRN17)

PRN17:
Doy=077, Transm. time: 64818 sec

![Graph showing orbit error](image)

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7.800000000000E+01 5.059375000000E+01 4.506973447820E-09 2.983492318682E+00
-9.257976353169E-05 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
4.176000000000E+05-5.401670932770E-08 4.040348681654E-01 7.636845111847E-08
9.60363015702E-01 2.215312500000E+02 2.547856603060E+00 7.964974630307E-09
-3.771585673111E-10 1.000000000000E+00 1.575000000000E+03 0.000000000000E+00
2.000000000000E+00 0.000000000000E+00 1.024454832077E-08 7.800000000000E+01
4.104180000000E+05 4.000000000000E+00

diff EPH.dat.org EPHcuc_x0.dat

---

"< -2.579763531685E-06 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03
> -9.257976353169E-05 5.277505260892E-03 8.186325430870E-06 5.153578153610E+03"

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Orbit error $\vec{\varepsilon}$

Range error

Baseline: $b=31.3\text{km}$

$$\delta\rho = -\vec{\varepsilon}^T \left( I - \hat{\rho} \cdot \hat{\rho}^T \right) \frac{b}{\rho}$$

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Exercise:

Justify that clock errors completely cancel in differential positioning.
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If the distance between the user and the reference station is "short enough", so that the residual error ionospheric, tropospheric and ephemeris are small compared to the typical errors due to receiver noise and multipath, it can be assumed:

\[ T_{ru}^j = I_{ru}^j = 0 ; \hat{\rho}_{0ru}^j \cdot \epsilon_{eph}^j = 0 \]

Note that the previous definition of “shortness” is quite fussy

Working with smoothed code, a residual error of about 0.5 metres could be tolerable, but for carrier based positioning it should be less than 1 cm to allow the carrier ambiguity fixing.

- The differential ephemeris error is at the level of few centimetres for baselines up to 100 km (i.e. 5 cm assuming a large bound of \( \epsilon_{eph}^j \leq 10 \) m).
- The typical spatial gradient of the ionosphere (STEC) is 1-2 mm/km (i.e. 0.1-0.2 m in 100km), but it can be more than one order of magnitude higher when the ionosphere is active.
Error mitigation and short baseline concept

Note that the previous definition of “shortness” is quite fussy

- The correlation radio of the troposphere is lower than for the ionosphere. At 10km of separation the residual error can be up to 0.1-0.2 m. Nevertheless, 90% of the tropospheric delay can be modelled and the remaining 10% can be estimated together with the coordinates (for high precision applications). For distances beyond a ten of kilometres or significant altitude difference it would be preferable to correct for the tropospheric delay at the reference station and user receiver.

**Carrier-smoothed code:**
Pseudorange code measurement errors due to receiver noise and multipath can be reduced smoothing the code with carrier measurements. Smoothed codes of 0.5m (RMS) can be obtained with 100 seconds smoothing. On the other hand, the ionospheric error is substantially eliminated in differential mode and the filter can be allowed for larger time smoothing windows.
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Differential code based positioning

If the reference station coordinates are known at the centimetre level and the distance between reference station and user are “not too large”, we can assume

\[ T_{ru}^j \approx 0; \quad I_{ru}^j \approx 0 \]
\[ \hat{\rho}_{0ru}^j \cdot \varepsilon_{eph}^j \approx 0 \]
\[ \varepsilon_{site}^j \approx 0 \implies \Delta r_{ru}^j \approx \Delta r_u^j \]

Thence,

\[ P_{ru}^j = \rho_{ru}^j + c \delta t_{ru}^j + I_{ru}^j + K_{ru}^j + \nu_{p ru}^j \]
\[ \rho_{ru}^j = \rho_{0ru}^j - \hat{\rho}_{0u}^j \cdot \Delta r_{ru}^j - \hat{\rho}_{0ru}^j \cdot \varepsilon_{site}^j + \hat{\rho}_{0ru}^j \cdot \varepsilon_{eph}^j \]

Note: for baselines up to 100 km the range error of broadcast orbits is less than 10 cm (assuming \( \varepsilon_{eph}^j \approx 10 \text{ m} \)).

The left hand side of previous equation can be spitted in two terms: one associated to the reference station and the other to the user:

\[ P_{ru}^j - \rho_{0ru}^j = -\hat{\rho}_{0u}^j \cdot \Delta r_u^j + c \delta t_{ru}^j + K_{ru}^j + \nu_{p ru}^j \]

Or, what is the same:

\[ P_{ru}^j - \rho_{0ru}^j = P_u^j - \rho_{0u}^j - \left( P_r^j - \rho_{0r}^j \right) \]

\[ \Delta r_{ru} = \Delta r_u - \Delta r_r = \Delta r_u - \varepsilon_{site} \]
Differential code based positioning

\[ P^j_{ru} - \rho^j_{0ru} = -\hat{\rho}^{j}_{0u} \cdot \Delta r_u + c \delta t_{ru} + K_{ru} + \nu^j_{pru} \]

The left hand side of previous equation can be spitted in two terms: one associated to the reference station and the other to the user:

\[ P^j_{ru} - \rho^j_{0ru} = P^j_u - \rho^j_{0u} - \left( P^j_r - \rho^j_{0r} \right) \]

- The term \( P^j_r - \rho^j_{0r} \) is the error in range measured by the reference station, which can be broadcasted to the user as a differential correction:

\[ PRC^j = \rho^j_{0r} - P^j_r \]

- The user applies this differential correction to remove/mitigate common errors:

\[ P^j_u - \rho^j_{0u} + PRC^j = -\hat{\rho}^{j}_{0u} \cdot \Delta r_u + c \delta \tilde{t}_{ru} + \nu^j_{pru} \]

Where the receiver’s instrumental delay term \( K_{ru} \) is included in the differential clock \( c \delta \tilde{t}_{ru} \)

For distances beyond a ten of kilometres, or significant altitude difference, it would be preferable to correct for the tropospheric delay at the reference station and user receiver.
The **reference station** with known coordinates, computes pseudorange and range-rate corrections: \( PRC = \rho_{\text{ref}}, -P_{\text{ref}}, \quad RRC = \Delta PRC/\Delta t \).

The **user** receiver applies the PRC and RRC to correct its own measurements, \( P_{\text{user}} + (PRC + RRC (t-t_0)) \), removing SIS errors and improving the positioning accuracy.

**DGNSS with code ranges**: users within a hundred of kilometres can obtain **one-meter-level** positioning accuracy using such pseudorange corrections.
Differential code based positioning

The user applies this differential correction to remove/mitigate common errors:

\[
P_u^j - \rho_{0u}^j + PRC^j = -\hat{\rho}_{0u}^j \cdot \Delta r_u + c \tilde{\delta}t_{ru} + \nu_{pru}^j
\]

where the receiver’s instrumental delay term \( K_{ru} \) is included in the differential clock \( c \tilde{\delta}t_{ru} \)

The previous system for navigation equations is written in matrix notation as:

\[
\begin{bmatrix}
\text{Pref}^1 \\
\text{Pref}^2 \\
\vdots \\
\text{Pref}^n
\end{bmatrix} = \begin{bmatrix}
-(\hat{\rho}_{0u}^1)^T & 1 \\
-(\hat{\rho}_{0u}^2)^T & 1 \\
\vdots & \vdots \\
-(\hat{\rho}_{0u}^n)^T & 1
\end{bmatrix} \begin{bmatrix}
\Delta r_u \\
c \tilde{\delta}t_{ru}
\end{bmatrix}
\]

where

\[
\text{Pref}^j \equiv P_u^j - \rho_{0u}^j + PRC^j
\]
Differential code based positioning

**Time synchronization issues:**

For simplicity we have dropped any reference to measurement epochs, but real-time implementations entail delays in data transmission and the time update interval can be limited by bandwidth restrictions.

- Differential corrections vary slowly and its useful life can be up to several minutes with S/A=off.

- To reduce bandwidth, the reference station computes Pseudorange Corrections (PRC) and Range-Rate Correction (RRC) for each satellite in view, which are broadcast to every several seconds, up to a minute interval with S/A=off.

- The user computes the PRC at the measurement epoch as:

\[
PRC^j(t) = PRC^j(t_0) + RRC^j(t - t_0)
\]
Differential code based positioning

Data handling:

Reference station and user have to coordinate how the measurements are to be processed:

- Corrections must be identified with an Issue of Data (IOD) and time-out must be considered.
- Both receivers must use the same ephemeris orbits (which are identified by the IODE).
- If reference station uses a tropospheric model the same model must be applied by the user.
- If reference station uses the broadcast ionospheric model, the user must do the same.

Note: we have considered here only code measurements. The carrier based positioning will be treated next, using double differences of measurements and targeting the ambiguity fixing.
The reference station with known coordinates, computes pseudorange and range-rate corrections: $PRC = \rho_{\text{ref}} - P_{\text{ref}}, \quad RRC = \Delta PRC / \Delta t$.

The user receiver applies the PRC and RRC to correct its own measurements, $P_{\text{user}} + (PRC(t_0) + RRC(t-t_0))$, removing SIS errors and improving the positioning accuracy.

*DGNSS with code ranges*: users within a hundred of kilometres can obtain one-meter-level positioning accuracy using such pseudorange corrections.
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ftp://cddis.gsfc.nasa.gov/highrate/2013/

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>USN3</td>
<td>39°12’19.0&quot;N</td>
<td>76°36’37.4&quot;W</td>
<td>23.6 km</td>
<td>76 m</td>
<td></td>
</tr>
<tr>
<td>GODS</td>
<td>39°12’19.0&quot;N</td>
<td>76°36’37.4&quot;W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GODN</td>
<td>39°12’19.0&quot;N</td>
<td>76°36’37.4&quot;W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USN3</td>
<td>39°12’19.0&quot;N</td>
<td>76°36’37.4&quot;W</td>
<td>23.6 km</td>
<td>76 m</td>
<td></td>
</tr>
</tbody>
</table>

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2. Augmentation Systems
   2.1. Introduction
   2.2. Ground-Based Augmentation system (GBAS)
   2.3. Satellite based Augmentation system (SBAS)
Introduction: What Augmentation is?

- To enhance the performance of the current GNSS with additional information to:
  - Improve INTEGRITY via real-time monitoring
  - Improve ACCURACY via differential corrections
  - Improve AVAILABILITY and CONTINUITY

- Satellite Based Augmentation Systems (SBAS)
  - E.g., WAAS, EGNOS, MSAS

- Ground Based Augmentation Systems (GBAS)
  - E.g., LAAS

- Aircraft Based Augmentation (ABAS)
  - E.g., RAIM, Inertials, Baro Altimeter
Why Augmentation Systems?

- Current GPS/GLONASS Navigation Systems cannot meet the Requirements for All Phases of Flight:
  - Accuracy
  - Integrity
  - Continuity
  - Availability

- Marine and land users also require some sort of augmentation for improving the GPS/ GLONASS performances.
WHY GNSS NEEDS AN AUGMENTATION?

<table>
<thead>
<tr>
<th>Performance</th>
<th>CATEGORY I Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>H. 13 m</td>
<td>V. 22m</td>
</tr>
<tr>
<td>99% (RAIM)</td>
<td>ACCURACY (95%)</td>
</tr>
<tr>
<td>99%</td>
<td>AVAILABILITY</td>
</tr>
<tr>
<td>?</td>
<td>INTEGRITY</td>
</tr>
<tr>
<td>?</td>
<td>CONTINUITY OF SERVICE</td>
</tr>
<tr>
<td>H. 16.0 m</td>
<td>V. 4.0 m</td>
</tr>
<tr>
<td>99% to 99.990%</td>
<td></td>
</tr>
<tr>
<td>2.10^-7/ approach</td>
<td></td>
</tr>
<tr>
<td>Time to alarm 6 s</td>
<td></td>
</tr>
<tr>
<td>10^-5 / approach</td>
<td></td>
</tr>
<tr>
<td>(10^-6 / 15 s)</td>
<td></td>
</tr>
</tbody>
</table>

GPS Only

Civil Aviation

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**Accuracy:** Difference between the measured position at any given time to the actual or true position.

Even with S/A off a Vertical Accuracy < 4m 95% of time cannot be guaranteed with the standalone GPS.

---

**ANALYSIS NOTES**
- Data taken from Overlook PAN Monitor Station, equipped with Trimble SVeeSix Receiver
- Single Frequency Civil Receiver
- Four Satellite Position Solution at Surveyed Benchmark
- Data presented is raw, no smoothing or editing

---

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**Integrity**: Ability of a system to provide timely warnings to users or to shut itself down when it should not be used for navigation.

Standalone GPS and GLONAS Integrity is Not Guaranteed

**GPS/GLONASS Satellites:**
Time to alarm is from minutes to hours
No indication of quality of service

**Health Messages:**
- GPS up to 2 hours late
- GLONASS up to 16 hours late
**Continuity:** Ability of a system to perform its function without (unpredicted) interruptions during the intended operation.

**Availability:** Ability of a system to perform its function at initiation of intended operation. System availability is the percentage of time that accuracy, integrity and continuity requirements are met.

---

**Availability and Continuity Must meet requirements**

---

**Continuity:**
Less than $10^{-5}$ Chance of Aborting a Procedure Once it is Initiated.

**Availability:**
$>99\%$ for every phase of flight (SARPS).
INTEGRITY

Less than $10^{-7}$ probability of true error larger than confidence bound. Time to alarm 6 s
SBAS and GBAS Navigation Modes

SBAS

WAAS

Enroute Oceanic

Enroute Domestic

Terminal

Approach

Surface

LAAS

GBAS
## Civil Aviation Signal-in-Space Performance Requirements

<table>
<thead>
<tr>
<th>Aviation</th>
<th>Accuracy (H) 95%</th>
<th>Accuracy (V) 95%</th>
<th>Alert Limit (H)</th>
<th>Alert Limit (V)</th>
<th>Integrity</th>
<th>Time to alert</th>
<th>Continuity to alert</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENR</td>
<td>3.7 Km (2.0 NM)</td>
<td>N/A</td>
<td>7400 m 3700 m 1850 m</td>
<td>N/A</td>
<td>1-10^-7/h</td>
<td>5 min.</td>
<td>1-10^-4/h to 1-10^-8/h</td>
<td>0.99 to 0.99999</td>
</tr>
<tr>
<td>TMA</td>
<td>0.74 Km (0.4 NM)</td>
<td>N/A</td>
<td>1850 m</td>
<td>N/A</td>
<td>1-10^-7/h</td>
<td>15 s</td>
<td>1-10^-4/h to 1-10^-8/h</td>
<td>0.999 to 0.99999</td>
</tr>
<tr>
<td>NPA</td>
<td>220 m (720 ft)</td>
<td>N/A</td>
<td>600 m</td>
<td>N/A</td>
<td>1-10-7/h</td>
<td>10 s</td>
<td>1-10-4/h to 1-10-8/h</td>
<td>0.99 to 0.99999</td>
</tr>
<tr>
<td>APV-I</td>
<td>220 m (720 ft)</td>
<td>20 m (66 ft)</td>
<td>600 m</td>
<td>50 m</td>
<td>1-2x10^-7 per approach</td>
<td>10 s</td>
<td>1-8x10^-6 in any 15 s</td>
<td>0.99 to 0.99999</td>
</tr>
<tr>
<td>APV-II</td>
<td>16.0 m (52 ft)</td>
<td>8.0 m (26 ft)</td>
<td>40 m</td>
<td>20 m</td>
<td>1-2x10^-7 per approach</td>
<td>6 s</td>
<td>1-8x10^-6 in any 15 s</td>
<td>0.99 to 0.99999</td>
</tr>
<tr>
<td>CAT-I</td>
<td>16.0 m (52 ft)</td>
<td>6.0 - 4.0 m (20 to 13 ft)</td>
<td>40 m</td>
<td>15 -10 m</td>
<td>1-2x10^-7 per approach</td>
<td>6 s</td>
<td>1-8x10^-6 in any 15 s</td>
<td>0.99 to 0.99999</td>
</tr>
</tbody>
</table>
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Most of the measurement errors are common: clock, ephemeris, ionosphere and troposphere.

A common correction valid for any receiver within the LADGPS area is generated and broadcast.

The accuracy is limited by the spatial decorrelation of those error sources (1m at 100Km).
GBAS Concept

The Ground Station (GS) is responsible for generating and broadcasting carrier-smoothed code differential corrections and approach-path information to user aircrafts. This system is used to support aircraft operations during approach and landing.

- It is also responsible of detecting and alarming space-segment and ground-segment failures.
- The GS must insure that all ranging sources for which GBAS corrections are broadcast are safe to use. If a failure occurs that threatens user safety, the GS must detect and alert users (by not broadcasting corrections for the affected ranging source) within a certain time-to-alert.

This is from [RD-5]
GBAS Components

The GBAS station involves redundant Reference Receivers and antennas, redundant High Frequency Data Broadcast equipment linked to a single antenna.

- The GS tracks, decodes, and monitors GPS satellite information and generates differential corrections.
- It also performs integrity checks on the generated corrections.
- The correction message, along with suitable integrity parameters and approach path information, is then broadcast to airborne users on a VHF channel, up to about 40km.

This is from [RD-5]
A variety of integrity monitor algorithms that are grouped into:

- Signal Quality Monitoring (SQM)
- Data Quality Monitoring (DQM)
- Measur. Quality Monitoring (MQM)
- Multiple Reference Consist Check (MRCC)
- $\sigma_\mu$-monitor
- Message Field Range Test (MFRT)
**SQM:** Targets satellite signals anomalies and local interference. Implements tests for Correlation Peak Symmetry, Receiver Signal Power and Code-Carrier Divergence.

**DQM:** Checks the validity of the GPS ephemeris and clock data for each satellite that rises in view of the LGF and at each time new navigation data messages are broadcast.

**MQM:** Confirms the consistency of the pseudorange and carrier-phase measurements over the last few epochs to detect sudden step and any other rapid errors.

**MRCC:** Examines the consistency of corrections for each satellite across all reference receivers.

**σμ-monitor:** Helps ensure a Gaussian distribution for the correction error with zero mean and that the broadcast $\sigma_{pr\_ gnd}$ overbounds the actual errors in the broadcast differential corrections.

**MFRT:** Verifies that the computed averaged pseudorange corrections and correction rates fit within the message field bounds.

**Executive Monitors (EXM-I and EXM-II)** coordinate all previous monitors and combine failure flags.
Horizontal position error: rov1: 2013 077 [0:85400] seconds

Vertical position error: rov1: 2013 077 [0:85400] seconds

2013_077_rov1 [gRS, 1.0s] N=86041

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The pseudorange error is split in its components.

- Clock error
- Ephemeris error
- Ionospheric error
- Local errors (troposphere, multipath, receiver noise)

Uses a network of receivers to cover broad geographic area
## Error Mitigation

<table>
<thead>
<tr>
<th>Error component</th>
<th>GBAS</th>
<th>SBAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite clock</td>
<td>Common Mode Differencing</td>
<td>Estimation and Removal each error component</td>
</tr>
<tr>
<td>Ephemeris</td>
<td></td>
<td>Fixed Model</td>
</tr>
<tr>
<td>Ionosphere</td>
<td></td>
<td>Carrier Smoothing by user</td>
</tr>
<tr>
<td>Troposphere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multipath and Receiver Noise</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Three existing SBAS Systems
The European Geoestationary Navigation Overlay SERVICE (EGNOS)

- EGNOS is the European component of a Satellite Based Augmentation to GPS.

- EGNOS is being developed under the responsibility of a tripartite group:
  - The European Space Agency (ESA)
  - The European Organization for the Safety of Air Navigation (EUROCONTROL)
  - The Commission of the European Union.
ECAC Area
(ECAC: European Civil Aviation Conference)
EGNOS
- RIMS
- MCC
- NLES

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EGNOS ground segment is composed of the following stations/centres which are mainly distributed in Europe and are interconnected between themselves through a land network.

- **37 RIMS** (Ranging and Integrity Monitoring Stations) + seven being deployed: receive the satellite signals and send this information to the MCC centres.

- **4 MCC** (Master Control Centres) receive the information from the RIMS stations and generate correction messages to improve satellite signal accuracy and information messages on the status of the satellites (integrity). The MCC acts as the EGNOS system 'brain'.

- **6 NLES** (Navigation Land Earth Stations): they receive the correction messages from the CPFs for the upload of the data stream to the geostationary satellites and the generation of the GPS-like signal. This data is then transmitted to the European users via the geostationary Satellite

http://egnos-user-support.essp-sas.eu/egnos_ops/egnos_system/system_description/current_architecture
Master Control Center (MCC)

MCC is Subdivided into

– CCF (Central Control Facility)
  Monitoring and control EGNOS G/S
  Mission Monitoring and archive
  ATC I/F

– CPF (Central Processing Facility)
  Provides EGNOS WAD corrections
  Ensures the Integrity of the EGNOS users
  Utilises independent RIMS channels for checking of corrections
  Real time software system developed to high software standards

4 MCCs are implemented in EGNOS
SBAS Differential Corrections and Integrity: The RTCA/MOPS-DO 229C
Message Format

- 250 bits
- One Message per second
- All messages have identical format

The corrections, even for individual satellites are distributed across several individual messages.
### SBAS Broadcast Messages (ICAO SARPS)

<table>
<thead>
<tr>
<th>MSG</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Don't use this SBAS signal for anything (for SBAS testing)</td>
</tr>
<tr>
<td>1</td>
<td>PRN Mask assignments, set up to 51 of 210 bits</td>
</tr>
<tr>
<td>2 to 5</td>
<td>Fast corrections</td>
</tr>
<tr>
<td>6</td>
<td>Integrity information</td>
</tr>
<tr>
<td>7</td>
<td>Fast correction degradation factor</td>
</tr>
<tr>
<td>8</td>
<td>Reserved for future messages</td>
</tr>
<tr>
<td>9</td>
<td>GEO navigation message (X, Y, Z, time, etc.)</td>
</tr>
<tr>
<td>10</td>
<td>Degradation Parameters</td>
</tr>
<tr>
<td>11</td>
<td>Reserved for future messages</td>
</tr>
<tr>
<td>12</td>
<td>SBAS Network Time/UTC offset parameters</td>
</tr>
<tr>
<td>13 to 16</td>
<td>Reserved for future messages</td>
</tr>
<tr>
<td>17</td>
<td>GEO satellite almanacs</td>
</tr>
<tr>
<td>18</td>
<td>Ionospheric grid point masks</td>
</tr>
<tr>
<td>19 to 23</td>
<td>Reserved for future messages</td>
</tr>
<tr>
<td>24</td>
<td>Mixed fast corrections/long term satellite error corrections</td>
</tr>
<tr>
<td>25</td>
<td>Long term satellite error corrections</td>
</tr>
<tr>
<td>26</td>
<td>Ionospheric delay corrections</td>
</tr>
<tr>
<td>27</td>
<td>SBAS outside service volume degradation</td>
</tr>
<tr>
<td>28 to 61</td>
<td>Reserved for future messages</td>
</tr>
<tr>
<td>62</td>
<td>Internal Test Message</td>
</tr>
<tr>
<td>63</td>
<td>Null Message</td>
</tr>
</tbody>
</table>

Many Message Types Coordinated Through Issues Data (IOD)
Issues of Data (IOD)
Message Time-Outs:
Users can operate even when missing Messages

- Prevents Use of Very Old Data
- Confidence Degrades When Data is Lost
- IODF: Detect Missing Fast Corrections

The Correction is estimated by the master station

**tof**: Time of applicability (1st bit of message)

Last bit of message: tof + 1 sec

Correction time-Out

System Latency
<table>
<thead>
<tr>
<th>Data</th>
<th>Associated Message Types</th>
<th>Maximum Update Interval (seconds)</th>
<th>En Route, Terminal, NPA Timeout (seconds)</th>
<th>Precision Approach Timeout (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAAS in Test Mode</td>
<td>0</td>
<td>6</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>PRN Mask</td>
<td>1</td>
<td>60</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>UDREI</td>
<td>2-6, 24</td>
<td>6</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>Fast Corrections</td>
<td>2-5, 24</td>
<td>60</td>
<td>(*)</td>
<td>(*)</td>
</tr>
<tr>
<td>Long Term Corrections</td>
<td>24, 25</td>
<td>120</td>
<td>360</td>
<td>240</td>
</tr>
<tr>
<td>GEO Nav. Data</td>
<td>9</td>
<td>120</td>
<td>360</td>
<td>240</td>
</tr>
<tr>
<td>Fast Correction Degradation</td>
<td>7</td>
<td>120</td>
<td>360</td>
<td>240</td>
</tr>
<tr>
<td>Weighting Factors</td>
<td>8</td>
<td>120</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>Degradation Parameters</td>
<td>10</td>
<td>120</td>
<td>360</td>
<td>240</td>
</tr>
<tr>
<td>Ionospheric Grid Mask</td>
<td>18</td>
<td>300</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Ionospheric Corrections</td>
<td>26</td>
<td>300</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>UTC Timing Data</td>
<td>12</td>
<td>300</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Almanac Data</td>
<td>17</td>
<td>300</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

(*) Fast Correction Time-Out intervals are given in MT7 [between 12 to 120 sec]
### PRN MASK (MT01)

<table>
<thead>
<tr>
<th>Bit No</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>.</th>
<th>38</th>
<th>.</th>
<th>120</th>
<th>.</th>
<th>210</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
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<td>1</td>
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<td>PRN</td>
<td>GPS PRN 2</td>
<td>GPS PRN 4</td>
<td>GPS PRN 5</td>
<td>GLONASS Slot 1</td>
<td>AORE PRN 120</td>
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<td>3</td>
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<td>29</td>
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</tr>
</tbody>
</table>

Each MT01 contains its associated IODP

Up to 51 satellites in 210 slots.

**Note:** Each Correction set in MT 2-5,5,6,7,24,25 its characterized by its PRN-Mask number, between 1 to 51.

<table>
<thead>
<tr>
<th>PRN Slot</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-37</td>
<td>GPS/GPS Reserved</td>
</tr>
<tr>
<td>38-61</td>
<td>GLONASS</td>
</tr>
<tr>
<td>62-119</td>
<td>Future GNSS</td>
</tr>
<tr>
<td>120-138</td>
<td>GEO/SBAS</td>
</tr>
<tr>
<td>139-210</td>
<td>Future GNSS/GEO/SBAS/Pseudolites</td>
</tr>
</tbody>
</table>
Example of message: Fast Corrections (MT2-5,24)

- Primarily Removed SA
  - Common to ALL users
  - Up to 13 Satellites Per Message
  - Pseudorange Correction /confidence Bound
  - Range Rate Formed by Differencing
  - UDRE degrades Over Time
    - Acceleration Term in MT 7
    - Reset when new Message Received
Example of message: Fast Corrections (MT2-5,24)

\[ PRC(t) = PRC_n + RRC_n(t - t_n) \]

\[ RRC_n = \frac{PRC_n - PRC_o}{t_n - t_o} \]

\[ Y = C1 + PRC - \rho^* + \Delta t_{sat} + dt_{sat} - TGD + IONO + TROP \]

DIRECTION OF DATA FLOW FROM SATELLITE; MOST SIGNIFICANT BIT (MSB) TRANSMITTED FIRST

- IODP (2 BITS)
- 250 BITS - 1 SECOND
- REPEAT FOR 12 MORE SATELLITES

- IODF (2 BITS)
- 13 12-BIT FAST CORRECTIONS
- 13 4-BIT UDREIs
- 24-BITS PARITY

13 12-BIT FAST CORRECTIONS

13 4-BIT UDREIs

REPEAT FOR 12 MORE SATELLITES

6-BIT MESSAGE TYPE IDENTIFIER (= 2, 3, 4 & 5)

8-BIT PREAMBLE OF 24 BITS TOTAL IN 3 CONTIGUOUS BLOCKS

\[ \sigma^2_{i,flt} = \sigma^2_{UDRE} + \epsilon^2_{fc} + \epsilon^2_{rrc} + \epsilon^2_{ltc} + \epsilon^2_{er} \]

\[ (RSS_{UDRE} = 0 \ [MT10]) \]
Example of message: Ionospheric Corrections (MT26)

• Only Required for Precision Approach
  – Grid of Vertical Ionospheric Corrections
  – Users Select 3 or 4 IGPs that Surrounding IPP
    • $5^\circ \times 5^\circ$ or $10^\circ \times 10^\circ$ for $55^\circ < \text{Lat} < 55^\circ$
    • Only $10^\circ \times 10^\circ$ for $55^\circ < |\text{Lat}| < 85^\circ$
    • Circular regions for $|\text{Lat}| > 85^\circ$
  – Vertical Correction and UIVE Interpolated to IPP
  – Both Converted to Slant by Obliquity Factor

IGP: Ionospheric Grid Point
IPP: Ionospheric Pierce Point
Global IGP Grid

Predefined Global IGP Grid

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Ephemeris + Clocks → Fast Corrections + Long Term Corrections + UDRE + Degradation Param.

IONO → IONO Corrections + GIVE + Degradation Param.

\[ y = C1 + PRC - \rho^* + \Delta t^{sat} + dt^{sat} - TGD - IONO - TROP \]

\[ \sigma^2 = \sigma_{flt}^2 + \sigma_{UIRE}^2 + \sigma_{air}^2 + \sigma_{tropo}^2 \]
### Navigation System Error and Protection levels

\[
x = [\Delta N, \Delta E, \Delta U, cdt]
\]

\[
P_x = (G^T W G)^{-1} = \begin{bmatrix}
    d_N^2 & d_{NE} & d_{NV} & d_{NT} \\
    d_{NE} & d_E^2 & d_{EV} & d_{ET} \\
    d_{NV} & d_{EV} & d_V^2 & d_{VT} \\
    d_{NT} & d_{ET} & d_{VT} & d_T^2
\end{bmatrix}
\]

\[
W = P_y^{-1}
\]

\[
P_y = \begin{bmatrix}
    \sigma_i^2 & 0 \\
    0 & \sigma_N^2
\end{bmatrix}
\]

\[
\sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,air}^2 + \sigma_{i,tropo}^2
\]

\[
HPL = 6.00 \sqrt{\frac{d_N^2 + d_E^2}{2}} + \sqrt{\left(\frac{d_N^2 - d_E^2}{2}\right)^2 + d_{NE}^2}
\]

\[
VPL = 5.33 \, d_V
\]

\[Z \sim \mathcal{N}(0,1)\]

\[P(|Z| > 5.33) = 10^{-7}\]
Fast and Long-Term Correction Degradation

\[
\sigma_{i,flt}^2 = \begin{cases} 
\left( \sigma_{UDRE} + \varepsilon_{fc} + \varepsilon_{rrc} + \varepsilon_{ltc} + \varepsilon_{er} \right)^2, & \text{if } RSS_{UDRE} = 0 \quad (MT10) \\
\sigma_{UDRE}^2 + \varepsilon_{fc}^2 + \varepsilon_{rrc}^2 + \varepsilon_{ltc}^2 + \varepsilon_{er}^2, & \text{if } RSS_{UDRE} = 1 \quad (MT10)
\end{cases}
\]

\[\varepsilon_{fc} = a \left( \frac{t-t_u+t_{lat}}{2} \right)^2\]

\[\varepsilon_{ltc,v0} = C_{ltc,v0} \text{ floor} \left( \frac{t-t_{ltc}}{I_{ltc,v0}} \right)\]

\[\varepsilon_{ltc,v1} = \begin{cases} 
0, & \text{if } t_u = t_{of} \\
C_{ltc,ltc} + C_{ltc,v1} \max \left\{ 0, t_0-t, t-t_0-I_{ltc,v1} \right\}, & \text{otherwise}
\end{cases}\]

\[\varepsilon_{er} = \begin{cases} 
0, & \text{Neither fast nor long term corrections have time out for precision approach} \\
C_{er}, & \text{Otherwise}
\end{cases}\]

\[\varepsilon_{rrc} = \begin{cases} 
0, & \text{IODOF \_current \_IODOF \_previous = 3} \\
\left( a \frac{I_{fc}}{4} + B_m \frac{\Delta t}{4} \right) (t-t_{of}), & \text{(IODOF \_current - IODOF \_previous) mod 2 = 1}
\end{cases}\]
Degradation of Ionospheric Corrections

\[ \sigma_{UIRE}^2 = F_{pp}^2 \sigma_{UIVE}^2 \]

\[ F_{pp} = \left[ 1 - \left( \frac{R_e \cos E}{R_e + h_i} \right)^2 \right]^{\frac{1}{2}} \]

\[ \sigma_{UIVE}^2 = \sum_{n=1}^{N} W_n \left( x_{pp}, y_{pp} \right) \sigma_{n,ionogrid}^2, \quad N = 4 \text{ or } 3 \]

\[ \sigma_{ionogrid}^2 = \begin{cases} 
(\sigma_{GIVE} + \varepsilon_{iono})^2, & \text{if } \text{RSS}_{iono} = 0 \quad (MT10) \\
\sigma_{GIVE}^2 + \varepsilon_{iono}^2, & \text{if } \text{RSS}_{iono} = 1 \quad (MT10) 
\end{cases} \]

\[ \varepsilon_{iono} = C_{iono \_step \_floor} \left( \frac{t - t_{iono}}{I_{iono}} \right) + C_{iono \_ramp} \left( t - t_{iono} \right) \]

MT10

- \( B_{rrc}, C_{ltc \_lsb}, C_{ltc \_v1}, \)
- \( I_{ltc \_v1}, C_{ltc \_v0}, I_{ltc \_v0}, \)
- \( C_{er}, \text{RSS}_{UDRE}, \)
- \( C_{iono \_ramp}, C_{iono \_step}, \)
- \( I_{iono}, \text{RSS}_{iono} \)

MT26

- \( t_{iono}, GIVE_i \)
Users know the receiver-satellites geometry and can compute bounds on the horizontal and vertical position errors. These bounds are called Protection Levels (HPL and VPL). They provide good confidence (10^{-7}/hour probability) that the true position is within a bubble around the computed position.

\[ VPL = K_v \sqrt{\sum_{i=1}^{N} S_{V_i}^2 \sigma_i^2} \]

\[ \sigma_i^2 = \sigma_{i,flt}^2 + \sigma_{i,UIRE}^2 + \sigma_{i,air}^2 + \sigma_{i,tropo}^2 \]

P(VPE>VPL) < 10^{-7} /sample
DGNSS implementations: WADGNSS (SBAS)
EGNOS

Accuracy

Integrity

Continuity

Availability

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References


<table>
<thead>
<tr>
<th>Error sources (1σ)</th>
<th>GPS - Error Size (m)</th>
<th>EGNOS - Error Size (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS SREW</td>
<td>4.0&lt;sup&gt;12&lt;/sup&gt;</td>
<td>2.3</td>
</tr>
<tr>
<td>Ionosphere (UIVD error)</td>
<td>2.0 to 5.0&lt;sup&gt;13&lt;/sup&gt;</td>
<td>0.5</td>
</tr>
<tr>
<td>Troposphere (vertical)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>GPS Receiver noise</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>GPS Multipath (45° elevation)</td>
<td>0.2</td>
<td>0.2</td>
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<tr>
<td>GPS UERE 5° elevation</td>
<td>7.4 to 15.6</td>
<td>4.2 (after EGNOS corrections)</td>
</tr>
<tr>
<td>GPS UERE 90° elevation</td>
<td>4.5 to 6.4</td>
<td>2.4 (after EGNOS corrections)</td>
</tr>
</tbody>
</table>

<sup>12</sup> GPS Standard Positioning Service Performance Standard [RD-3].

<sup>13</sup> This is the typical range of ionospheric residual errors after application of the baseline Klobuchar model broadcast by GPS for mid-latitude regions.

SREW: Satellite Residual Error for the Worst user location.
UIVD: User Ionospheric Vertical Delay.
UERE: User Equivalent Range Error