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Measuring geocentric radial coordinates with a non-fiducial GPS network

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Abstract. In this paper we address the problem of estimating the short term precision of the geocentric radial coordinate of a GPS receiver placed on the Earth crust using a non-fiducial approach. The network used in our analysis contains 35 receivers distributed globally. We have analyzed the data with two different strategies: global and regional. In the global strategy the results obtained, which are compatible with those of Heflin et al. (1992) and Blewitt et al. (1992), provide a weighted root mean square of the residuals (*wrms*) one order of magnitude larger than the formal errors of the individual estimates. Our regional strategy is based on the assumption that errors in the orbit determination induce errors in the receiver positions, correlated up to large scales. This approach allows us to obtain a significant agreement between the wrms and the formal errors.

1. Introduction

For geodynamical applications, the GPS observables obtained using a network of GPS receivers can be analyzed in a great variety of ways (see, for example, Blewitt, 1993). In the fiducial approach the coordinates of a subset of stations are constrained to some apriori values, sometimes derived from other techniques like Very Long Baseline Interferometry or Laser Ranging. Some disadvantages of this approach are a) the possible errors in the measurement of the link between the GPS receivers and the other instruments inducing undesired distortions in the results, b) the impossibility of studying the fiducial stations themselves and c) the reduction of the chances of making comparisons between different techniques. In the non-fiducial approach the mentioned constraints are removed. In this case, the disadvantage is the complete correlation between rotations describing the orientation of the reference systems involved. A solution investigated by Heflin et al (1992) and Blewitt et al. (1992) is to use a "seven parameter" transformation to "align" the independent solutions obtained analyzing data from a global network of 21 stations during 21 consecutive days with a non-fiducial strategy. Blewitt et al. conclude that this approach produces rms differences between GPS and ITRF90 of 12 mm for receivers placed in northern hemisphere. In this paper, also in coherence with the *Occam's razor*, we have chosen the non-fiducial approach.

We present here an analysis of the data gathered during a 5-day period (from 15 to 19 November 1993) using a network composed of the 35 Rogue GPS receivers listed in Fig. 1 and Table 1. This is a subset of the network of 41 Rogue GPS Receivers operated by the International GPS Service for Geodynamics (IGS) at this time. The analysis is concentrated on this 5-day period, in which data was also collected at 16 other GPS receivers located close to tide gauges in Great Britain, France, Spain and Portugal. These receivers were deployed as part of the Eurogauge Project for the study of vertical land displacements and mean sea level on the Atlantic Coast of Europe (Ashkenazi et al, 1994 and Torres et al, 1994).

2. General approach

We consider GPS data sets, gathered with a network of receivers $\Re$, observing a constellation of GPS satellites S during N disjoint time intervals ending at epochs $\tau = \tau_1 \dots \tau_N$. In our case we have used data obtained in $N = 5$ consecutive days, ending at epochs τ corresponding to the 24:00 of each day. The results to be obtained could be summarized using two sets of functions $\vec{x}_\tau$ and $\vec{y}_\tau$, which assign, for each τ, estimates of the geocentric cartesian coordinates to each receiver r and to each satellite s in suitable reference frames:

$$\vec{x} = \vec{x}_\tau(r)$$
$$\vec{y} = \vec{y}_\tau(s)$$

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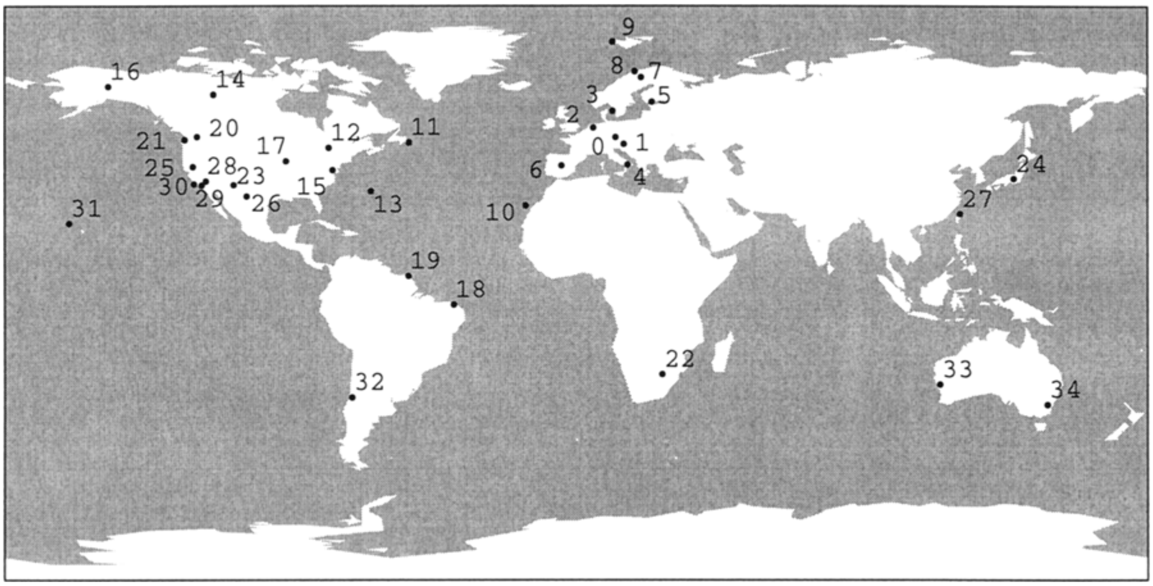

Fig. 1. Global network of 35 stations used in the computations of days 15-19 November 1993. The sequence follows increasing distances from Wettzell (number 0). The station identifiers and names are indicated in Table 1.

Imperfections in the description of the observations in terms of the different models involved will produce imperfect estimates of $\vec{x}_\tau$ and $\vec{y}_\tau$. The different causes will produce errors at different spatial scales. Examples of error sources acting on local (significant correlation at distances less than 100 km), regional (less than 5000 km) or global scales are:

1. the unmodelled delays experienced by the signals when crossing the troposphere and the ionosphere, which are expected to be in a large part uncorrelated between receivers separated by distances greater than 100 km.
2. the stochastic nature of the forces acting on the satellites that produce a bad determination of the satellite positions $\vec{y}_\tau(s)$.
3. the complete correlation between common variations in the orientation of the satellite orbits and the variation in the Earth orientation, produces estimates of the station coordinates and its covariance matrix which are not unique.

There are other possible sources of error but, in this study, we will assume that any of them could be classified either as local or as non-local (global or regional). This situation could be expressed by the equations:

$$\vec{x}_\tau(r) = \vec{x}^*_\tau(r) + \vec{\phi}_\tau(r) \quad (1)$$

and

$$\vec{y}_\tau(s) = \vec{y}^*_\tau(s) + \vec{\psi}_\tau(s)$$

where the functions $\vec{x}^*_\tau(r)$ and $\vec{y}^*_\tau(s)$ are local random processes and $\vec{\phi}_\tau(r)$ and $\vec{\psi}_\tau(s)$ are non-local random processes. $\vec{x}^*_\tau(r)$ and $\vec{y}^*_\tau(s)$ could be interpreted as the estimate of the *corrected* terrestrial and celestial geocentric coordinates of site r and satellite s respectively.

The observables, ignoring instrumental biases, are the satellite-receiver distances and they constrain the errors in the satellite and receiver coordinates, $\Delta\vec{x}_\tau(r)$ and $\Delta\vec{y}_\tau(s)$, approximately in the form:

$$(\vec{y}_\tau(s) - \vec{x}_\tau(r)) \cdot (\Delta\vec{y}_\tau(s) - \Delta\vec{x}_\tau(r)) \simeq 0 \quad (2)$$

This relation holds for any visible satellite at point r. Therefore from Eq. 2 we could estimate a displacement $\Delta\vec{x}_\tau(r)$ as a linear combination of the errors in all visible satellites at time τ:

$$\Delta\vec{x}_\tau(r) = \sum_s a_s(r)\Delta\vec{y}_\tau(s)$$

being $a_s(r)$ coefficients which depend on the station r and satellite s considered. Other receiver r' with simultaneous visibility of a large number of satellites will produce correlated values of $\Delta\vec{x}_\tau(r')$ and, therefore, it will contribute to build up $\vec{\phi}_\tau$. We will consider that our network is sufficiently dense to allow us to take $\vec{\phi}_\tau$ as a smooth function.

If $\Delta\vec{y}_\tau(s)$ are uncorrelated among different satellites with common dispersions, the autocorrelation of $\Delta\vec{x}_\tau(r)$ is:

$$R_\tau(r,r') \propto n(r,r')$$

where $n(r,r')$ is the corresponding number of common satellites simultaneously observed at receivers r, r'.

If we choose a reference station r_0 and a distance R, we could consider the subnetwork $\Re$ of receivers with distances to r_0 smaller than R (i.e. $\Re$ will have a *center* r_0 and a *radius* R). In Fig. 2 we can see that n reduces by half at distances of the order of 5000 km which we could take as the correlation radius of the errors in the receivers coordinates produced by the mismodelling of the orbits.

Our purpose is to separate the small and the large scale components of $\vec{x}_\tau(r)$, $\vec{x}^*_\tau(r)$ and $\vec{\phi}_\tau(r)$ respectively, and to measure its dispersion. We could assume the validity of the linear approximation inside a region centered on the receiver r_0, smaller than the correlation radius of $\vec{\phi}_\tau(r)$:

$$\vec{\phi}_\tau(r) = \vec{c}_\tau(r_0) + D_\tau(r_0) \cdot \vec{x}^*_\tau(r)$$

where $\vec{c}_\tau(r_0)$ is a vector and $D_\tau(r_0)$ is a linear application, superposition of a rotation –antisymmetrical part $H_\tau(r_0)$– and a deformation –symmetrical part $S_\tau(r_0)$–. The antisymmetrical part is an expression of the impossibility of determining absolute space orientations of the satellite and receivers based on GPS measurements. The symmetrical part accounts for the deformation introduced by the changes in the relative positions of the satellites. We will assume that this deformation is simply a scale factor $s_\tau(r_0)$:

$$D_\tau(r_0) = H_\tau(r_0) + s_\tau(r_0) \cdot I$$

where I is the identity mapping. If $\vec{c}_\tau(r_0)$, $H_\tau(r_0)$ and the scale factor $s_\tau(r_0)$ are independent of r_0 we will have the standard *seven parameter* or Helmert transformation. We have taken the mean value $< \vec{x}_\tau(r) >$ –weighted with the full covariance matrix– of $\vec{x}_{\tau_1}, \ldots, \vec{x}_{\tau_N}$ as the initial estimate of the coordinates of the receiver r. Because of $\vec{\phi}_\tau$ is a smooth function in a region that contains $\Re$, the transformation can be estimated using the approximate equations:

$$\vec{x}_\tau(r) =< \vec{x}_\tau(r) > + \vec{c}_\tau(r_0) + D_\tau(r_0) \cdot < \vec{x}_\tau(r) > \qquad (3)$$

where r is any receiver belonging to the set of receivers $\Re$ *centered* on r_0. In our case the selection of the subset $\Re$ is performed by increasing the values of the elements of the covariance corresponding to the receivers not included in the network.

Once we have obtained $\vec{c}^{\Re}_\tau(r_0)$ and $D^{\Re}_\tau(r_0)$, we can get the corrected values $\vec{x}^*_\tau$:

$$\vec{x}^{*\Re}_\tau(r) = \vec{x}_\tau(r) - \vec{c}^{\Re}_\tau(r_0) - D^{\Re}_\tau(r_0) \cdot < \vec{x}_\tau(r) > \qquad (4)$$

Analogously to the one-dimensional case (see Papoulis, 1965, p.335), the validity of this filtering process depends critically on whether or not the radius R of the subnetwork is significantly smaller than the correlation radius of $\vec{\phi}_\tau(r)$. If this condition is not adequately fulfilled, we will have an artificial increase in the noise of $\vec{x}^{*\Re}_\tau(r)$. Also, the estimates will depend on the choice of the reference coordinates in Eq. 3.

In the study of vertical land displacements and mean sea level we are interested in the radial component of the previous equation and the necessary equations are obtained by projecting Eq. 4 into the corresponding radial direction. For each receiver r, the *wrms* or *repeatebility* of the radial component is defined as

$$wrms(r) = \sqrt{\sum_{\tau=1}^{N} \frac{1}{\sigma^2_\tau(r)} \delta\rho^2_\tau(r) \Big/ \sum_{\tau=1}^{N} \frac{1}{\sigma^2_\tau(r)}}$$

where $\delta\rho_\tau(r)$ is the radial component of the deviation of $\vec{x}^{*\Re}_\tau(r)$ respects its weighted mean, and $\sigma^2_\tau(r)$ is the corresponding variance.

3. Results

We have analyzed the data using *GIPSY-II*, a software package developed at JPL for the analysis of GPS data (Lichten and Boder, 1987; Sovers and Border, 1990; Heflin et al.,1992; Heflin, 1993). Included in *GIPSY-II* is the capability of computing a station coordinate covariance matrix free from the effects derived from the complete correlation between changes in the orientation of the receivers and the satellites (see section *Constructing the Weight Matrix* in Blewitt et al., 1992).

The data set considered in these pages was collected during the first Eurogauge campaign, 15 to 19 November 1993, at the stations listed in Table 1 and shown in Fig. 1. The models used in the computations are the same as those of Heflin et al. (1992) and Blewitt et al. (1992). As it has been mentioned we have chosen the free-network or non-fiducial approach in order to a) exclude the possibility of contaminating our results because of wrong assumptions on the coordinates of the fiducial sites, and b) avoid the need of the inclusion of a *geocenter offset* (Blewitt et al. 1992, Malla et al. 1993).

In a first step we have used the complete network $\overline{\Re}$. The estimated solution corresponds to the case in which it is assumed that the translation, the rotation and the scale factor are constant for the whole Earth. In Figs. 3,4 the daily residuals with respect to the mean value of the radial component are plotted for each station. It is evident the positive correlations between the residuals of nearby stations, as those of Europe. This result suggests that the

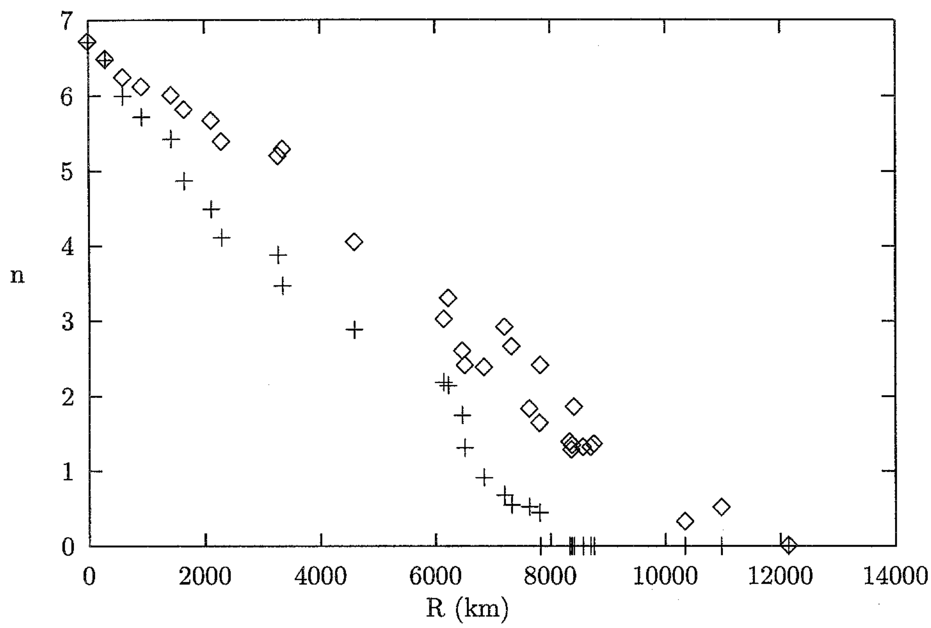

Fig. 2. Mean number of satellites simultaneously visible, n, as a function of the radius R in km, taking Wettzell as the reference, during the 17 November 1993 session. + corresponds to the satellites observed with a subnetwork and ◇ corresponds to the satellite observed with a pair of stations.

high values of wrms obtained compared with the formal error of the radial components $\rho_i^{\Re}$ (typically 2 cm) (see Table 1) are due to systematic effects as those discussed previously. The wrms of the radial component computed in this way ranges up to 10 cm, improving the results of Heflin et al. (1992) which showed a mean daily repeatebility of 15 cm.

In order to remove the spatial correlation effects we have also processed our data taking into account different subnetworks. The results presented correspond to the subnetworks centered at Wettzell. Equivalent results have been obtained centering the subnetworks on Quincy, which presents similar density of receivers. In Table 1 we have indexed the stations of the network according to its proximity to Wettzell. We define the subnetwork $\Re_i$ as the set of receivers with indices ranging from 0 to i. Applying the data analysis described previously, we have estimated $\vec{c}_T^{\Re_i}$, $H_T^{\Re_i}$ and $s_T^{\Re_i}$ and the wrms of the solution in the receiver radial components for each subnetwork $\Re_i$ with $i = 5 \ldots 34$. The estimated scale factor was small ($\approx 10^{-9}$) for all the subnetworks. The rotations have no significance in our analysis of the radial component and therefore we have ignored them. In Fig. 5 we have plotted the cartesian components of $\vec{c}_T^{\Re_i}$. It can be observed that their differences between different days can reach 20 cm. In some days there is also a significant dependence of these components with the subnetwork radius.

4. Discussion

In Fig. 6 we can observe that we get a smaller wrms for smaller radius. These variations can be a consequence of the filtering process which extracts the large scale contribution $\vec{\phi}_T$ in Eq. 1 has not the appropriate size. We obtain an improvement of one order of magnitude when we perform the transformation with subnetwork radius lesser than 3000 km, compatible with the described variation for the components of the translation showed in Fig. 5. This fact suggests that the main effect that degrades the wrms for subnetworks with large radius can be modelled as a random field $\vec{c}_T(r_0)$, affecting differently to stations with few visible-satellites in common (Fig. 2).

Because of seven parameters are needed to define $\vec{\phi}_T(r)$, the radius of the subnetwork should be large enough to contain at least 3 receivers. This competes with the need of using subnetworks significantly smaller than the correlation radius mentioned before. In our case the smaller network that we consider is $\Re_4$, containing Wettzell, Graz, Kootwijk, Onsala and Matera, with a radius of the order of 1000 km. The wrms obtained with this network are less than 5 mm (see Fig. 6), smaller than the formal errors.

Using subnetworks with radius bigger than approximately 4000 km produces an increase in the fluctuations of the estimations of $\vec{x}_T^*(r)$. This will, possibly, produce an incorrect separation between the different scale processes.

In particular the wrms obtained with the European scale subnetwork ($\Re_8$, Table 1) has a mean value of

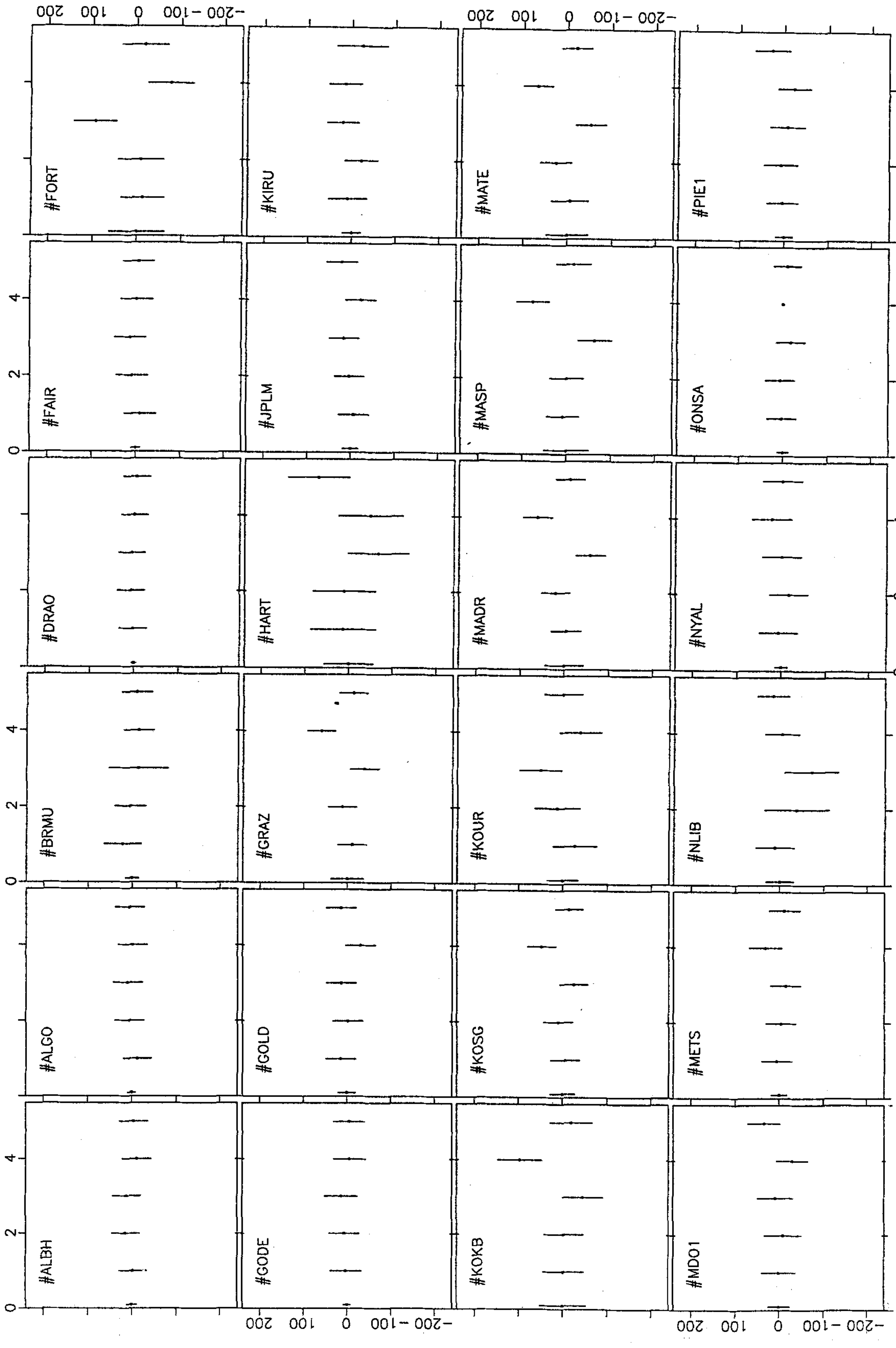

Fig. 3. Each box contains the daily residuals in mm (vertical axes) with respect to the mean value of the radial component, which are plotted for 24 of the 35 stations (label #XXXX) of the complete network R. For each box, the horizontal axes are the numbers 1 to 5 which correspond to the days 15 to 19 of November 1993. The *wrms* (mm) is also plotted on the label 0. Zero points with zero-length error bars indicate stations without data for the corresponding day (only Onsala during the 18 november 1993).

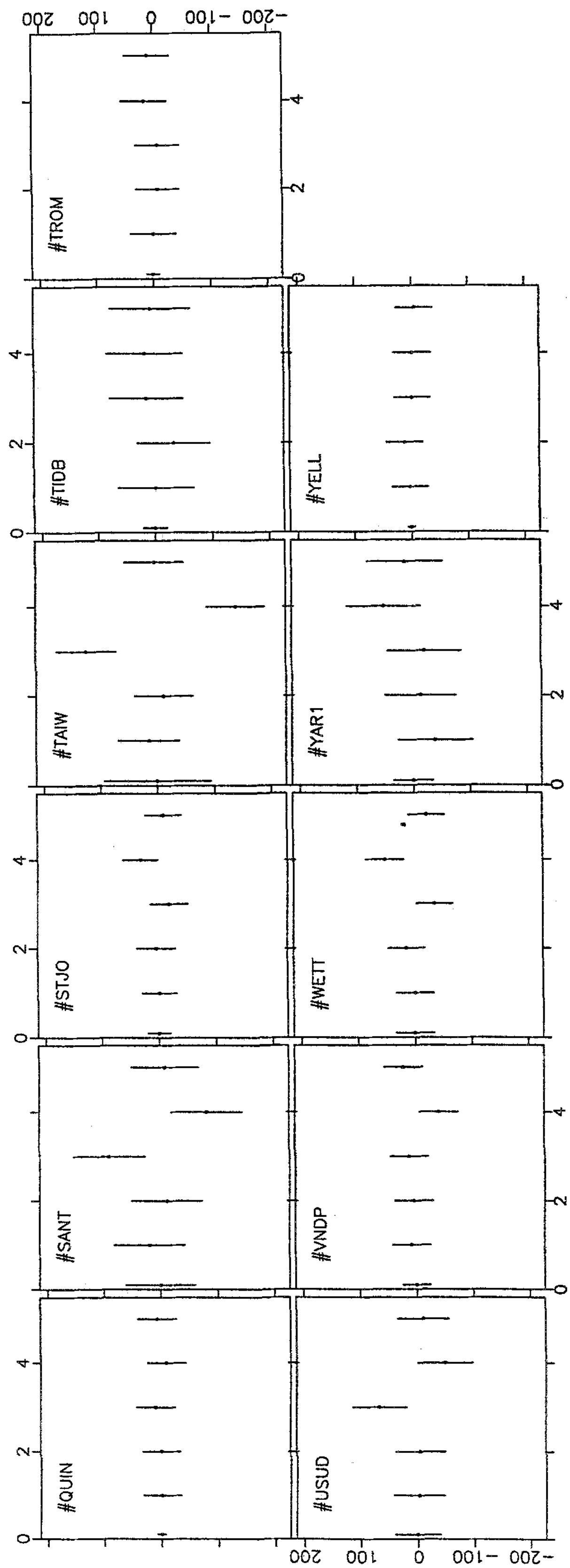

Fig. 4. Continuation of Figure 3 for the remaining 11 stations.

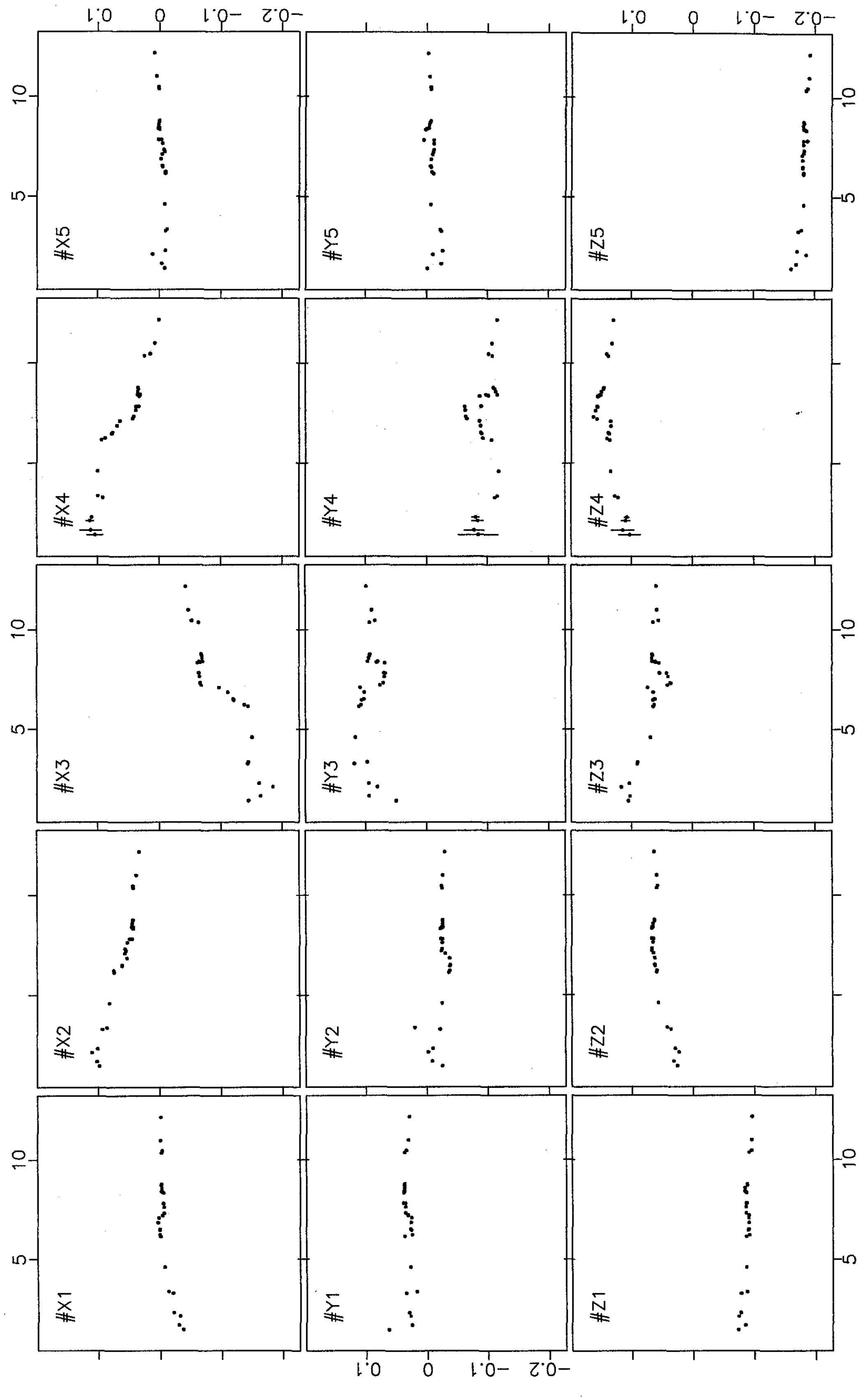

Fig. 5. The most significant parameters of the Helmert transformation, components X, Y and Z (m) of the translation $\mathbf{c}_i$ are plotted as a function of the subnetwork radius (units of horizontal axes in 10^3 km). The box labelled #Cd (C=X,Y,Z, d=1,2,3,4,5) contains the C component for day d

i	Sta.Id.	Sta.Name	d_{0i} (km)	$\rho_i^{\overline{\Re}}$ (m)	wrms$\rho_i^{\overline{\Re}}$ (m)	wrms$\rho_i^{\Re_8}$ (m)
0	WETT	WETTZELL	0	6366613.3	0.034	0.003
1	GRAZ	GRAZ	302	6367256.8	0.037	0.005
2	KOSG	KOOTWIJK	603	6364932.4	0.028	0.004
3	ONSA	ONSALA	920	6363044.0	0.012	0.002
4	MATE	MATERA	990	6369641.8	0.047	0.001
5	METS	METSAHOVI	1432	6362156.5	0.018	0.003
6	MADR	MADRID	1655	6370016.7	0.043	0.003
7	KIRU	KIRUNA	2123	6360203.2	0.021	0.012
8	TROM	TROMSO	2296	6359486.9	0.011	0.012
9	NYAL	NY ALESUND	3283	6357625.7	0.014	
10	MASP	MASPALOMAS	3361	6373725.8	0.051	
11	STJO	ST JOHN'S	4599	6366674.7	0.019	
12	ALGO	ALGONQUIN	6155	6367333.7	0.009	
13	BRMU	BERMUDA	6231	6372032.2	0.015	
14	YELL	YELLOWKNIFE	6473	6361528.7	0.006	
15	GODE	GSFC	6522	6369717.2	0.008	
16	FAIR	FAIRBANKS	6857	6360923.5	0.010	
17	NLIB	NORTH LIBERTY	7103	6368898.3	0.031	
18	FORT	FORTALEZA	7215	6378060.0	0.062	
19	KOUR	KOUROU	7333	6377933.6	0.035	
20	DRAO	PENTICTON	7640	6366423.2	0.005	
21	ALBH	ALBERT HEAD	7821	6366258.4	0.011	
22	HART	HARTEBEESTHOEK	7832	6375643.8	0.056	
23	PIE1	PIE TOWN	8339	6373732.0	0.018	
24	USUD	USUDA	8382	6372250.9	0.040	
25	QUIN	QUINCY	8385	6370459.9	0.008	
26	MDO1	MACDONALD	8418	6374608.3	0.024	
27	TAIW	TAI SHAI	8419	6374383.5	0.094	
28	GOLD	GOLDSTONE	8583	6371978.8	0.019	
29	JPLM	JPL MESA	8712	6371841.8	0.018	
30	VNDP	VANDENBERG	8766	6371284.4	0.024	
31	KOKB	KOKEE	10358	6376292.4	0.052	
32	SANT	SANTIAGO	10461	6372502.8	0.061	
33	YAR1	YARRAGADEE	10991	6373369.6	0.035	
34	TIDB	TIDBINBILLA	12157	6371666.6	0.021	

Table 1. List of the 35 GPS stations, processed with data gathered during the days 15-19 November 1993, in ascending order of distance to Wettzell, d_{0i}. Their approximate mean radial coordinate ρ_i and its wrms are indicated using the global and the european network ($\overline{\Re}$ and $\Re_8$ respectively).

5 ± 4 mm. This is our estimate of the quality of the results establishing the detection limit of radial coordinate variations at this scale. The next stage of our research is to include the data from the 16 *non-IGS* GPS receivers, deployed as part of the Eurogauge Project, which will increase the density of stations in Europe and, consequently, extending our results to smaller scales.

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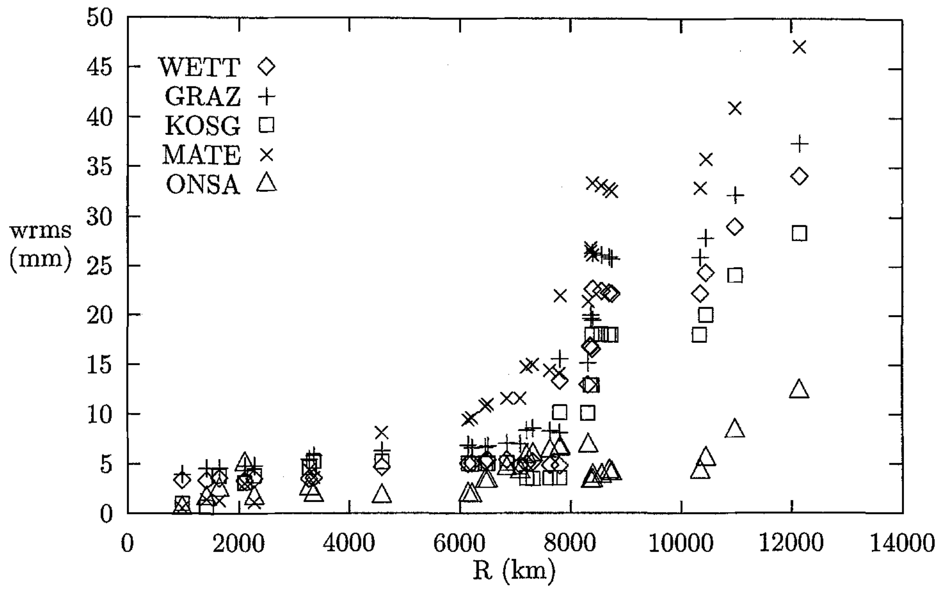

Fig. 6. The wrms of the radial position of the indicated sites are plotted vs. the radius R of subnetwork centered on Wettzell used in the analysis.

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