Available online at www.sciencedirect.com
ScienceDirect
Advances in Space Research 43 (2009) 1694–1701
Advances in Space Research
(a COSPAR publication)
www.elsevier.com/locate/asr

Obtaining more accurate electron density profiles from bending angle with GPS occultation data: FORMOSAT-3/COSMIC constellation

A. Aragon-Angel *, M. Hernandez-Pajares, J.M. Juan, J. Sanz

Research Group of Astronomy and GEomatics, Technical University of Catalonia, Jordi Girona 1-3 Module C3, 08034 Barcelona, Spain

Received 31 October 2007; received in revised form 28 October 2008; accepted 30 October 2008

Abstract

Since 1995, with the first GPS occultation mission on board Low Earth Orbiter (LEO) GPS/MET, inversion techniques were being applied to GPS occultation data to retrieve accurate worldwide distributed refractivity profiles, i.e. electron density profiles in the case of Ionosphere. Important points to guarantee the accuracy is to take into account horizontal gradients and topside electron content above the LEO orbit. This allows improving the accuracy from 20% to 50%, depending on the conditions, latitude and epoch regarding to Solar cycle as reported in previous works.

More recently, the satellite Constellation Observing System for Meteoroly Ionosphere and Climate (FORMOSAT-3/COSMIC), formed by 6 micro-satellites carrying a GPS receiver on board, is being deployed since April 2006 in circular orbit around the Earth, with a final altitude of about 700–800 km. Its global and almost uniform coverage will overcome the sparcity of data, one of the main limitations of other techniques providing direct observations of electron density profiles, such as ionosondes.

This new amount of incoming data can significantly stimulate the development of radio occultation techniques with the use of the huge volume of data provided by the FORMOSAT-3/COSMIC constellation to be processed and analysed updating the current knowledge of the Ionosphere.

In this context, a summary of the improved Abel transform inversion technique and the first results based on COSMIC constellation data will be presented. Moreover, comparison of different approaches and strategies in the occultation data inversion will be compared and discussed, taking advantage of the availability of FORMOSAT-3/COSMIC datasets. In particular, the use of two different observables as main input data for the radio occultation inversions will be analysed, implementing for the first time the improved Abel transform to bending angles derived from phase measurements. Furthermore, a new method for clock drift calibration will be also introduced.

Keywords: Radio occultations; FORMOSAT-3/COSMIC; GPS; Ionosphere; Electron density profiles; Bending angle

1. Introduction

Electron density profiles can be derived from occultation data. An occultation event occurs when a GPS satellite sets/rises below/above the horizon of a LEO satellite. Under such circumstances, when the LEO is equipped with a GPS receiver on board (which is the case of the FORMOSAT-3/COSMIC constellation, see Rocken et al. (2000)), the change in the delay and the bending of the signal path between the GPS and the LEO satellite caused by the atmosphere can be derived from the observations from the GPS receiver on the LEO (Hajj and Romans, 1998). This study represents a kick off to fine tune former developed tools by the authors and new implementations in order to analyse the vast amount of occultation data from the FORMOSAT-3/COSMIC constellation for the corresponding improvement, knowledge and applications.

* Corresponding author.
E-mail address: angela@ma4.upc.edu (A. Aragon-Angel).
URL: http://gage.es (A. Aragon-Angel).

0273-1177/$36.00 © 2009 COSPAR. Published by Elsevier Ltd. All rights reserved.
doi:10.1016/j.asr.2008.10.034

Two basic observables are mainly used as main datum to retrieve vertical ionospheric profiles: the linear combination of GPS dual frequencies $LI = L1 - L2$, the so-called geometric free combination (Parkinson and Spilker, 1996), and the extra Doppler shift induced by the medium in a single frequency (Schreiner et al., 1999). The use of one or another observable presents a different drawback. On one hand, when working with LI, it is implicitly assumed that $L1$ and $L2$ travel the same path but due to the dispersive nature of the ionosphere, $L1$ and $L2$ will follow slightly different paths. On the other hand, when working with the Doppler shift, it will be required to properly remove the non-ionospheric Doppler terms, in particular, the clock drifts of transmitter and receiver from the raw phase data. Note that all the results presented throughout the paper are referred to $L1$ frequency.

The new work presented is focused in the application of previously studied occultation techniques based on LI (see Hernández-Pajares et al., 2000; Garcia-Fernández et al., 2003) to FORMOSAT-3/COSMIC constellation, exploring as well the bending angle approach to these data, taking advantage of the new constellation availability. The bending approach can be used to study as well the neutral atmosphere, hence the importance to explore new implementations on it. Actually, the main intention of this paper is to proof that the implementation of the improved Abel inversion was possible also when using the bending angle as main observable, not only with the combination of phases LI as reported in previous works. It is going to be shown that it is possible and that would allow its extension to tropospheric heights in order to improve the neutral atmosphere refractivity retrieval. Therefore, at this point, we were more interested in proving the feasibility of such implementation. In this context it must be pointed out that the improved Abel transform inversion assumes that the electron density can be expressed as a function of a horizontal dependent function, given by the externally computed VTEC, and an unknown shape function, to be determined in the inversion process. In this sense, the gradients are different depending on horizontal and vertical coordinates. But of course, this is still a simplification (for instance, the separability hypothesis implies that the maximum electron density height – hmF2 – remains constant in the occultation region) but much more realistic than the classical Abel transform inversion assuming spherical symmetry for the electron density distribution.

2. Previous work: ionospheric carrier phase as main datum

In previous studies (Hernández-Pajares et al., 2000; Garcia-Fernández et al., 2003), the ionospheric GPS observations in occultation scenarios, basically ambiguous STEC (Slant Total Electron Content) values observed at negative elevations, have been used to estimate electron densities due to their high sensitivity to vertical variations of electron content. This electron content retrieval can be performed without any a priori solution:

(1) In a global framework combined with ground GNSS data. For instance, in Hernández-Pajares et al. (1998), a 3D voxel model of the ionosphere (tomographic model) were solved, by means of a Kalman filtering. A one-hour-update filter in a Sun-fixed reference frame, with a resolution of $10^\circ \times 10^\circ$ in latitude/local time and 100 km in height was considered. The information used as input data for the 3D voxel model was not only provided by a low orbiting GPS receiver, the GPS/MET in such study, both positive and negative elevation observations are used, but also from ground stations belonging to the International GPS Service IGS, with more than 100 ground GPS stations worldwide distributed.

(2) Or using just the single occultation data, by means of the classical Abel transform inversion. In this strategy, the data are processed independently for each occultation in order to achieve a better resolution with lower computational burden.

When following the second approach, and based on the definition of STEC as the line integral of electron density Ne, STEC can be expressed as the Abel transform of Ne (see Bracewell, 2000) under the assumption of *spherical symmetry* and neglecting the electron content above the LEO orbit:

$$STEC(p) = 2 \cdot \int_{l_0}^{l_{\rm LEO}} \frac{Ne(r)r}{\sqrt{r^2 - p^2}}\, dr \qquad (1)$$

where l_0 stands for the impact parameter at the beginning of the occultation, $l_{\rm LEO}$ the LEO position, p the impact parameter, and r the radius. The spherical symmetry assumption, which lays within the hypothesis of use of the classical Abel transform inversion as commented above, can be formulated in the case of electron density Ne as:

$$Ne(LT, LAT, H) = \phi(H) \qquad (2)$$

where LT stands for the longitude, LAT latitude and H height. The only assumed dependence of Ne is with respect to height, not latitude nor longitude. An equivalent method to reach the solution in Eq. (1) is by using a recursive process starting from the outer ray. At the ith step, the STEC corresponding to the ith impact parameter (p_i) would correspond to (see Hernández-Pajares et al., 2000):

$$STEC(p_i) = 2 \cdot l_{ii} \cdot Ne(p_i) + \sum_{j=1}^{j=i-1} 2 \cdot l_{ij} \cdot Ne(p_j) \qquad (3)$$

This approach presents mainly two missmodelings, such as *spherical symmetry* and neglected topside electron content contribution. They were respectively mitigated by means of the *Separability concept* and direct estimation. The *Separability concept* was integrated in the recursive procedure by considering not only the radial component (H) dependence but also longitude and latitude in the density calculation. Therefore, Ne can be reformulated as:

$$Ne(LT, LAT, H) = VTEC(LT, LAT) \cdot F(H) \quad (4)$$

where $VTEC(LT, LAT)$ represents the Vertical Total Electron Content (VTEC) at (LT, LAT) location and $F(H)$ stands for the so-called shape function, which assumes the height dependance. Actually, it is a simple but very effective approach: VTEC is considered as a good approximation to describe the horizontal variability of the electron density Ne. Under this separability assumption, the new estimated value is the shape function F instead of the electron density itself. Considering the latidudinal and logitudinal variation of VTEC into the electron density description, specially when studing a ionospherically disturbed area, it improves the quality of the retrieved electron density profiles. The benefits of implementing the Separability hypothesis were stated when comparing with the actual ionosonde data (see Hernández-Pajares et al., 2000; Garcia-Fernández et al., 2003). These comparisons confirmed the improvement in the electron density estimation Ne: a RMS reduction of 25–35% (20% in disturbed ionosphere) in Solar Minimum, and 35–50% in Solar Maximum. The improvement in electron density estimates reaches up to 40% in the E layer and around 50% in the E sporadic Es layer.

3. Working with Doppler shift as main datum

Electron density profiles can be also retrieved applying the classical Abel transform inverse to the bending angles derived from the atmospheric induced Doppler shift in $L1$ (see Hajj and Romans, 1998) in occulting scenarios (see sketch representation in Fig. 1). In order to compute accurate radio occultation inversions, the clock drifts of the GPS transmitter and receiver clocks should be removed from the raw phase data in order to solve the bending angles derived from the Doppler $L1$ phase excess. Otherwise, a completely unrealistic result (several orders of magnitude higher) would be obtained as depicted in Fig. 2. This can be done either by:

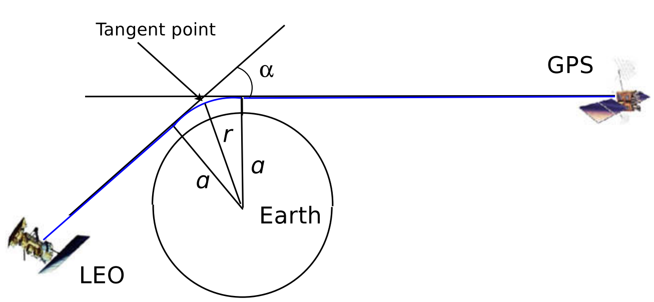

Fig. 1. Occultation geometry showing the bending of the signal (α) due to the dispersive nature of the atmosphere where a corresponds to the impact parameter and r to the radius at the tangent point. Typical bending for refractive indexes at tropospheric heights.

(1) Working with double-differences regarding to a ground station in common viewing of the occulting LEO and GPS, and another non-occulting LEO.
(2) Subtracting the excess Doppler of the ionospheric-free combination Lc of carrier phases from $L1$ excess Doppler (see Fig. 3, blue), which is the basic observable in this case. Note that there is the problem of different ray path between $L1$ and Lc observables. This procedure is only valid for ionospheric heights hence the discrepancies showed below 60 km (troposheric delay signature in red).

The latter is only valid for ionospheric heights while the former is valid for neutral atmosphere as well. Thanks to the FORMOSAT-3/COSMIC constellation configuration, a six evenly distributed LEO network of satellites, it is possible to use a second non-occulting LEO satellite to perform the double differences as suggested on Rocken et al. (2000) since complete double difference coverage is provided by the new constellation (see Fig. 3, red). Up to now, the double differencing would have involved a ground station, but there is the drawback of the potential presence of high frequency multipath when using ground based GPS data to remove clock drifts (see Ogaja and Satirapod, 2007).

The basic observable is the phase path (expressed in meters):

$$L = \int_{\text{GPS}}^{\text{LEO}} n\, ds \quad (5)$$

where L stands for either $L1$ or $L2$ carrier phase observables, n refraction index and the integral extends from the LEO position up to the GPS one. From the phase path, the excess phase is defined as:

$$\Delta L = L - |\vec{r}_{\text{LEO}} - \vec{r}_{\text{GPS}}| \quad (6)$$

The observable needed is the phase change, the so-called excess Doppler or Doppler shift:

$$f_d = \frac{d\Delta L}{dt} \quad (7)$$

The Doppler shift at both the transmitter and the receiver is produced by the atmospheric and ionospheric refraction index change, after subtracting the velocities of both, transmitter and receiver, projected along the actual signal propagation directions.

As already pointed out, the signal Doppler shift f_d becomes the fundamental observable. The Doppler shift of the operating frequency f_T can be derived using:

$$f_d = \frac{f_T}{c}\left(\vec{v_T} \cdot \widehat{e_T} + \vec{v_R} \cdot \widehat{e_R}\right) = -\frac{f_T}{c}\left(v_T^r \cos\phi_T + v_T^\theta \sin\phi_T + v_R^r \cos\phi_R - v_R^\theta \sin\phi_R\right) \quad (8)$$

where c is the speed of light, $\vec{v_T}$ and $\vec{v_R}$ are the transmitter and receiver velocities, $\widehat{e_T}$ and $\widehat{e_R}$ are the unit vectors in the

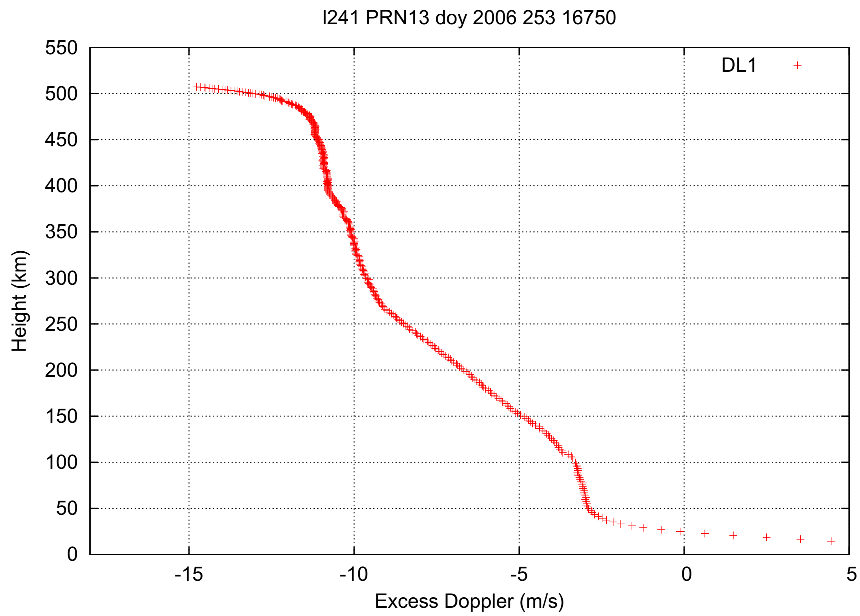

Fig. 2. FORMOSAT-3/COSMIC occultation: PRN 17 (where PRN stands for GPS satellite identification number), day 253 of 2006, 12h;41m approx. It can be seen the unrealistic excess Doppler in $L1$ observable without clock drift removal.

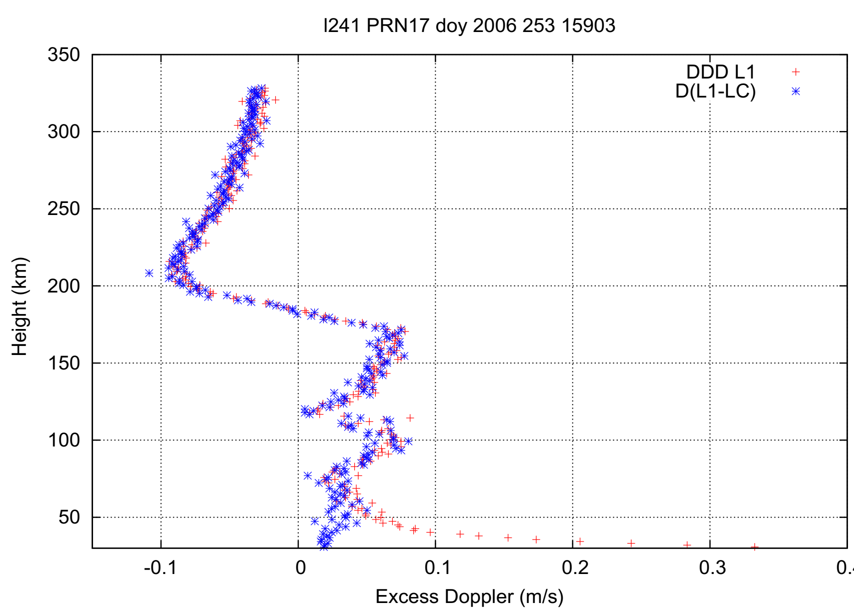

Fig. 3. FORMOSAT-3/COSMIC occultation: PRN 17 (where PRN stands for GPS satellite identification number), day 253 of 2006, 12h41m approx. Remaining observable after clock drift removal: with plus symbols, double differencing using a second non-occulting LEO satellite and, with asterisk symbols, subtracting the Lc combination to L1. An agreement of both approaches is shown at ionospheric heights, as expected, while the double differencing (red) curve shows the remaining tropospheric bending at heights below 60 km, which does not cancel out in this case. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)

direction of the transmitted and received signal, v_T^r and v_T^θ represent the radial and azimuthal components of the transmitting spacecraft velocity and, respectively, v_R^r and v_R^θ for the LEO receiver.

The signal path is curved according to Snell's law due to the changes in the index of refraction along the signal path. By assuming a spherical symmetric medium, Snell's law is replaced by Bouger's law leading to an extra constraint for the system to be solved:

$$n(\vec{r_t}) \cdot \left\| \vec{r_t} \times \widehat{k_t} \right\| = n(\vec{r_r}) \cdot \left\| \vec{r_r} \times \widehat{k_r} \right\| \quad (9)$$

where $\vec{r_t}$ and $\vec{r_r}$ represent the coordinates of the transmitter and receiver,n is the refraction index at the specified coordinates, and $\widehat{k_t}$ and $\widehat{k_r}$ are the unit vectors in the direction of the straight line connecting the transmitter to the receiver. To obtain the total atmospheric bending, Eqs. (8 and 9) are solved simultaneously. However, the knowledge of n at $\vec{r_t}$ and $\vec{r_r}$ is required. To overcome this issue, in a first iteration, the following approximation is made, which overestimates the electron density not more than 0.5% (Hajj and Romans, 1998).

$$n(\vec{r_t}) = n(\vec{r_r}) = 1 \tag{10}$$

Actually, the higher the altitude of the LEO is, the more reasonable the approximation in Eq. (10) becomes. In a spherical symmetric medium, the bending of the signal can be related to the index of refraction by means of the following integral:

$$\alpha(a) = -2a \int_0^\infty \frac{1}{\sqrt{a'^2 - a^2}} \frac{dln(n)}{da'} da' \tag{11}$$

whrere α stands for the bending angle, a for the impact parameter and n, the refractive index. By using an Abel integral transform, Eq. (11) can be inverted (see Tricomi, 1985), obtaining the refraction index as a function of the impact parameter a:

$$ln(n(a)) = -\frac{1}{\pi} \int_a^\infty \frac{\alpha(a')}{\sqrt{a'^2 - a^2}} da' \tag{12}$$

The upper limit of the integral in Eq. (12) requires knowledge of the bending α as function of a up to the top of the atmosphere. For practical matters, the bending angles above the LEO can either be neglected, extrapolated somehow or replaced by a climatological model (Schreiner et al., 1999). In the current study, this integral is solved up to the LEO height, hence the bending angles above the LEO orbit are neglected. Moreover, in the current approach, Eq. (11) is discretized and a recursive solution for the refractive index n is obtained starting from the outer ray inwards (see Fig. 4):

$$\alpha_i = \sum_{j=1}^{i-1} \varphi_j^{\mathrm{LEO}} + \sum_{j=1}^{i-1} \varphi_j^{\mathrm{GPS}} - \tan\varphi_i \cdot \frac{n_i - n_{i-1}}{n_{i-1}} \tag{13}$$

$$\alpha_j^{\mathrm{receiver}} = -\tan\varphi_j \cdot \frac{n_j - n_{j-1}}{n_{j-1}} \tag{14}$$

where α_j^{LEO} and α_j^{GPS} represent the bending angle of the LEO satellite, respectively the GPS satellite, at the jth layer and φ_i is the angle at the ith layer intersection with radial vector from the Earth's center to the GPS satellite, respectively the LEO satellite. At that point, and in order to solve electron densities from refractive index, the following relationship is used, valid for GPS frequencies (see Parkinson and Spilker, 1996):

$$n - 1 = \frac{-40.3 \cdot Ne}{f^2} \tag{15}$$

where f stands for the carrier frequency of the transmitted signal in Hz ($L1$ in this case), and Ne is given in e/m^3. Eq. (15) only considers main terms in the dependency of the Earth's ionosphere on electron density Ne (ions and the Earth's magnetic field are neglected).

The new contribution to occultation techniques is the implementation of the *Separability concept* to Eq. (15), that is to say, to substitute Ne by the expression given in Eq. (4) and solving for the new unknown, the shape function. Nevertheless, the approximation in Eq. (15) is not enough to linearize the problem when solving the refraction index under the *Separability hypothesis*. An extra approximation is needed regarding the working system frequency versus the plasma frequency, which rarely exceeds 20 MHz. The latter is neglected to obtain the final linearized expression relating n and Ne at two consecutive layers:

$$\frac{n_j - n_{j-1}}{n_{j-1}} \propto Ne_j - Ne_{j-1} = \Delta Ne_j \tag{16}$$

From this expression, and substituting Eq. (4), electron density profiles can be derived with separability implemented to bending angle. A general overall of the global procedure in provided in Fig. 5. This method could allow its extension to neutral atmosphere, which is not possible with occultations derived from LI observable.

4. First results using FORMOSAT-3/COSMIC data

The FORMOSAT-3/COSMIC constellation provides global observations of refractivity, pressure, temperature, humidity, TEC, ionospheric electron density, ionospheric scintillation climate monitoring, geodetic research. As already stated, the recent deployment of such constellation has opened new opportunities not only to test different

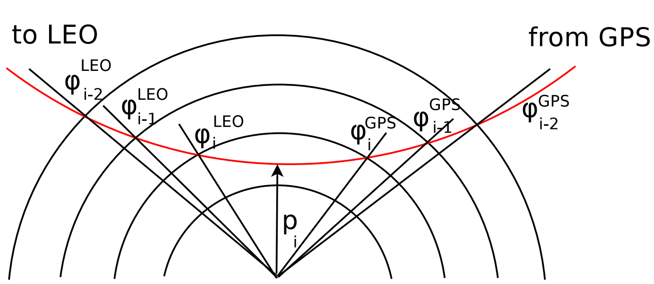

Fig. 4. Recursive solution starting from the outer ray. φ_i is the angle between the radial and the ray propagation directions for each layer. Both φ_i^{GPS} and φ_i^{LEO} contribute to the total bending angle α_i of the ray with impact parameter p_i. The bending of the signal in this figure would correspond to bending at ionospheric heights and has been exaggerated.

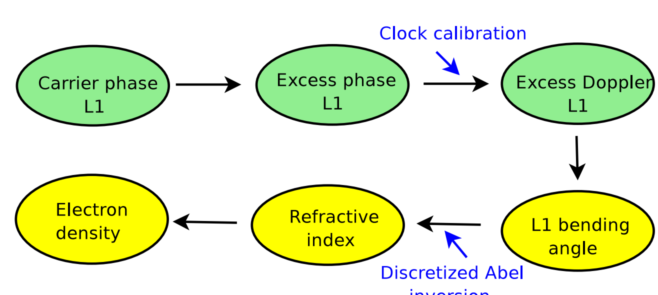

Fig. 5. Overall methodology to calculate the input observable (bending angle) from measurements (carrier phase) corrected from clock drifts and application of discritized Abel inversion as described in Fig. 4. and Eq. (14).

electron density retrieval aspects (old and new implementations) in order to improve radio occultation techniques but also the vast amount of worldwide electron density profiles, about 2500 per day, in front of 500 or 250 approximately per day from previous missions.

One first result from this study is regarding the excess ionospheric phase calibration, already presented in the previous section. Classically, the clock drift has been removed by double differencing using a fiducial site on the Earth's surface. The suggested double differences by means of a second LEO satellite (Rocken et al., 2000) instead of a ground site have been implemented. Nevertheless, subtracting the Lc observable to the main input data, $L1$, is the one finally implemented once their equivalence at ionospheric heights has been shown (see an example in Fig. 3). It has been shown that the Abel transform from ionospheric carrier phase observable, LI, and from $L1$ bending angle are quite equivalent, providing similar electron density profiles (see Fig. 6). This suggests that the difference in ray paths is not important (confirmed in comparisons and statistics performed in Aragon-Angel (2008)).

While both techniques are conceptually rather simple, one issue that has to be carefully considered is the detection and correction of measurement errors, for instance, cycle slips. Under carrier phase cycle slips, using the bending approach, it is easier to detect and correct cycle-slips than in the case of the LI method. For instance, in Fig. 7, both algorithms have been fed with the same input data, contaminated with one non-detected (hence, not repaired) cycle slip and the resulting profiles show the robustness of the bending approach. When using the bending angle, even under the presence of a cycle slip, since the geometry is not varying, the method is able to 'ingest' the cycle slip while when working with LI, a new bias should be calculated (the first observation when performing the recursive solution, is used to determine the bias of LI. If there is a cycle slip, the bias should be recalculated since the previous one is not valid any more, or equivalently the cycle-slip should be repaired). On the other hand, Abel inversion improved with the *Separability* approach can also provide significantly different results even under the present Solar Minimum conditions, compared with spherical symmetry, indicating the convenience of applying such improved technique in any part of the Solar cycle (see Fig. 8).

5. Conclusions and future work

Procedures to retrieve electron density profiles with high resolution and low computational burden, which were developed during previous LEO GPS occultation missions such as GPS/MET, SAC-C and CHAMP, have proven their validity with FORMOSAT-3/COSMIC constellation data as well. In particular, on one hand, it has been illustrated as main point the similarity of the results obtained from the ionospheric carrier phase combination LI versus the $L1$ bending angle procedure. On the other hand, the use of the *Separability hypothesis* in the Abel transform inversion, which models horizontal gradients, provides a significant improvement, even under Solar Minimum conditions. An example of the viability of its applicability to FORMOSAT-3/COSMIC constellation is given.

Moreover, it has been also pointed out the necessity to properly model the clock drift in the bending angle approach giving a new and easy way to remove it by means of subtracting the Lc combination to the carrier phase observable. This has shown that the different ray path between the observables does not affect the inversions. This

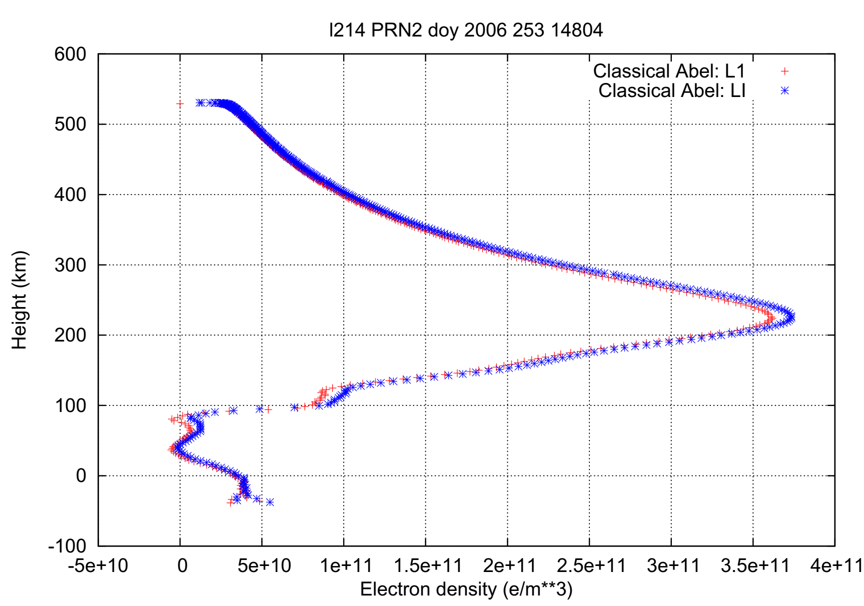

Fig. 6. FORMOSAT-3/COSMIC occultation: day 253 of 2006, FORMOSAT-3/COSMIC data, PRN 02 (where PRN stands for GPS satellite identification number), 04h27m UT approx. Abel tranform inverse from LI, and from $L1$ bending provide compatible electron density profiles.

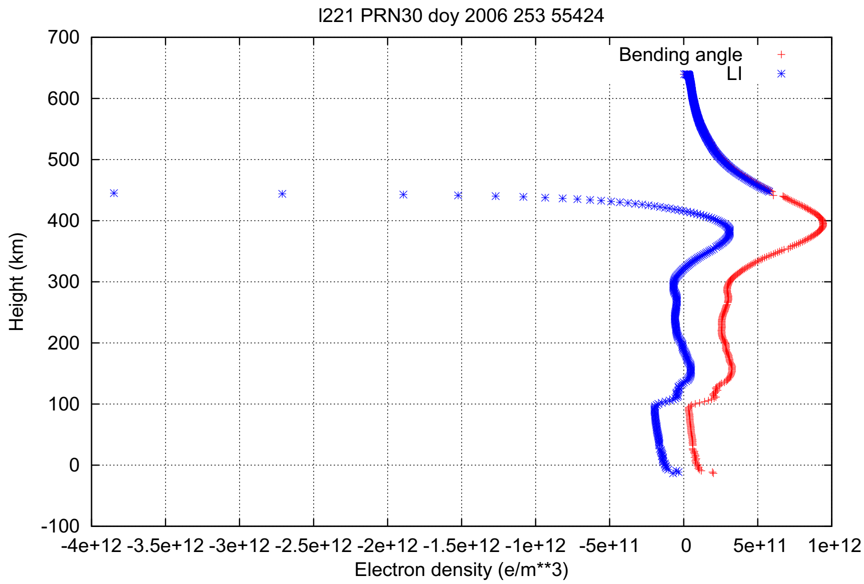

Fig. 7. Same occultation solved by using both *LI* and bending angle: the non-detected cycle slip does not represent a problem for solving the density profile using the bending angle observable.

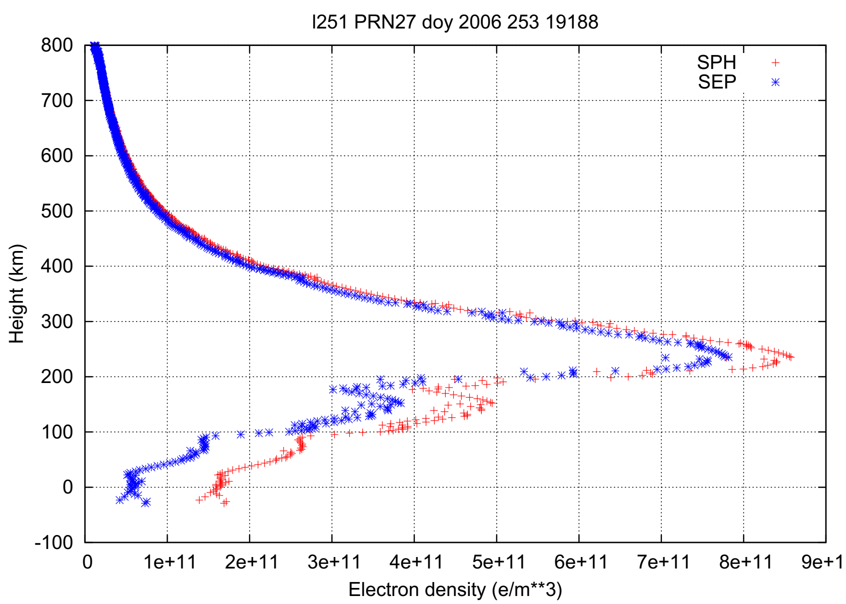

Fig. 8. FORMOSAT-3/COSMIC occultation: day 253 of 2006, FORMOSAT-3/COSMIC data, PRN 02 (where PRN stands for GPS satellite identification number), 04h27m UT approx. Improved Abel (with separability) applied to bending angle in blue versus ordinary bending angle approach (spherical symmetry). Note the significantly different results even under the present Solar Minimum conditions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)

clock drift calibration is needed, otherwise unrealistic excess phase data would be used as input data for the bending angle approach. As an added value to the bending approach, phase cycle slips are easier to detect and repair that working with *LI*. Future studies should perform an extensive characterization of the electron density distribution from FORMOSAT-3/COSMIC and study the feasibility and applicability of the *Separability* approach with the bending observable in order to improve the neutral atmospheric refractivity retrieval.

References

Aragon-Angel, M.A. New Technique to Improve the Electron Density Retrieval Accuracy: Application to FORMOSAT-3/COSMIC Constellation, ION GNSS 2008, 16th–19th September 2008, Savannah (Georgia), USA, 2008.

Bracewell, R.N. The Fourier Transform and its Applications, third ed McGraw-Hill, Boston, MA, 2000.

Garcia-Fernández, M., Hernández-Pajares, M., Juan, M., Sanz, J. Improvement of ionospheric electron density estimation with GPS-MET occultations using Abel inversion and VTEC information. J. Geophys. Res. 108 (A9), 1338, doi:10.1029/2003JA009952, 2003.

Hajj, G.A., Romans, L.J. Ionospheric electron density profiles obtained with the Global Positioning System: results from the GPS/MET experiment. Radio Sci. 33 (1), 175–190, 1998.

Hernández-Pajares, M., Juan, J.M., Sanz, J., Sole, J.G. Global observation of the ionospheric electronic response to solar events using ground and LEO GPS data. J. Geophys. Res. Space Phys. 61, 1237–1247, 1998.

Hernández-Pajares, M., Juan, J.M., Sanz, J. Improving the Abel inversion by adding ground data LEO radio occultations in the ionospheric sounding. Geophys. Res. Lett. 27, 27432746, 2000.

Ogaja, C., Satirapod, C. Analysis of High-Frequency Multipath in 1-Hz GPS Kinematic Solutions GPS Solutions. Springer, Berlin, Received: 25 September 2006, accepted: 6 February 2007, 2007.

Parkinson, B.W., Spilker Jr., J.J. Global Positioning System: Theory and Applications, Vols. 1 and 2, pp. 485–496, American Institute of Aeronautics, 370 L'Enfant Promenade, SW, Washington, DC, 1996.

Rocken, C., Kuo, Y.H., Schreiner, W., Hunt, D., Sokolovsky, S. COSMIC system description, special issue of terrestrial. Atmos. Ocean. Sci. 11 (1), 21–52, 2000.

Schreiner, W.S., Sokolovskiy, S.V., Rocken, C. Analysis and validation of GPS/MET radio occultation data in the ionosphere. Radio Sci. 34 (4), 949–966, 1999.

Tricomi, F.G. Integral Equations. Dover, New York, 1985.