Improvement of the Ionospheric Radio Occultation Retrievals by Means of Accurate Global Ionospheric Maps

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Abstract

The Abel inversion method is the classic approach to retrieve electron density profiles from radio occultation measurements, assuming that the electron density is only dependent upon height. In the early 2000s, the total electron content (TEC)-aided inversion method was introduced, where horizontal gradients in the electron density distribution are taken into account by using information from an external model of total electron content (TEC). We show how the quality of the chosen external ionospheric model influences the radio occultation retrievals. Using a two-layer TEC model, the 68% percentiles of the global error distributions of retrieved critical frequency and bottomside ionospheric electron content for 2014 improve more than 30%, with regard to the classic Abel inversion results. In a well-sounded region like Europe, this improvement raises to about 40–45%, while 10% improvement is achieved using two-layer maps with regard to the retrievals using single-layer TEC maps. We point out that using the TEC to describe the horizontal gradient incurs an implicit mismodeling that, until now, has not been taken into account. A new technique is presented that, thanks to TEC modeling by means of two layers, can assess the impacts of such mismodeling. From this technique, a method to select the most accurate radio occultation retrievals is applied to demonstrate that the resulting errors in the retrieved critical frequencies for the European region are smaller than 7% in nearly 82% of the cases, very similar to the relative difference between measurements of two nearby ionosondes.

1. Introduction

In the past century, Hajj and Romans (1998) and Schreiner et al. (1999) introduced the study of electron density in the ionosphere (Nₑ) by means of the Abel transform inversion for retrieving vertical profiles from GPS radio occultation (RO) observations. This Abel transform inversion technique is mainly based on three assumptions: (1) spherical symmetry, that is, the electron density is spherically symmetric and only dependent on height; (2) straight-line signal propagation between GPS and Low Earth Orbit (LEO) satellite links; and (3) an initial value of electron density at some top altitude sounded by the occultation. In the following, we will refer to this specific technique as classic approach.

Schreiner et al. (1999), it was shown that the spherical symmetry assumption is one of the major error sources in the electron density retrievals from RO and errors of approximately 20% were reported compared to retrievals with ionosonde measurements. These errors have latitudinal and longitudinal dependencies, especially in the case of low-latitude regions the presence of plasma caves (Lee et al., 2012) introduces large spatial gradients that cannot be accounted by the classic approach, giving rise to larger errors in low latitude than in the surrounding midlatitude regions (Chou et al., 2017; Habarulema et al., 2014).

In 1998, the International GNSS Service (IGS) started to compute global ionospheric maps (GIMs) of total electron content (TEC) based on observations gathered by a worldwide distributed network of ground receivers. Using those maps, horizontal gradients of TEC can be taken into account in the RO inversion process (TEC-aided Abel inversion). Different methods of this type have been proposed to improve the results of the classic approach (Aragon-Angel et al., 2010; Hernández-Pajares et al., 2000; Yue et al., 2013). More recently, regional or global maps of quantities, such as the F₂ peak electron density (NmF₂) or previously determined electron density profiles (EDPs), have been used to develop similar methodologies to improve the accuracy of RO retrievals (Chou et al., 2017; Pedatella et al., 2015a; Tulasi Ram et al., 2016). However, the NₑF₂ or Nₑ global maps used by those methodologies are derived after collecting data from EDPs calculated with the classic approach during a long period, typically one month in order to have enough spatial coverage (hereafter we will refer to this type of methods as iterative). These
iterative methods assume that the ionosphere is essentially stationary during that long time interval, which can be unrealistic. The situation will improve once the COSMIC-2 mission becomes operational, but in the meantime, GIMs from IGS have a lower latency, of only a few days and even less than 24 hr for the rapid product (Hernández-Pajares et al., 2009), and a higher spatial and temporal resolution than the maps proposed by the above-mentioned alternative strategies to improve the classic approach. IGS GIMs are based on precise GNSS ground measurements typically collected during a 24-hr period from a network of ground stations mostly distributed over continental regions, and have been demonstrated to achieve a global accuracy ranging from 3 to 10 TECU ($1 \text{ TECU} = 10^{16} \text{ e}^{-} \text{ m}^{-2}$) depending of the period of the year (Rovira-Garcia, Juan, Sanz, González-Casado, Ibáñez-Segura, 2016). The separability method (SM), introduced in Hernández-Pajares et al. (2000), is a TEC-aided Abel inversion method based on the hypothesis that $N_e$ can be written as the product of two functions, one being the TEC depending on the horizontal coordinates and the other depending only on the vertical height. Thanks to this strategy, the SM is able to calculate vertical EDPs, unlike previously mentioned methods that only calculate oblique EDPs. Using the SM, García-Fernández et al. (2003, 2005) and Aragon-Angel et al. (2010) compared the results of RO retrieved $N_e$ with ionosonde measurements of $N_{mF2}$ and reported average improvements of 30 and 45%, respectively, over the results obtained from the classic approach. However, in a subsequent study, using data from the Jicamarca ionosonde near the magnetic equator, Aragon-Angel et al. (2011) found a 10% improvement when using a narrow window to collocate the RO retrieved EDPs that were used in the comparison. More recently, Yue et al. (2013), when comparing with ionosonde measurements, found a quite moderate improvement, of about 7%, in their TEC-aided Abel inversion results with regard to the classic approach. One reason for the discrepancies in the results can be attributed to the lack of collocation (LoC) between ionosondes and ROs. This trend was already noted in García-Fernández et al. (2003) when using a maximum distance of 2,000 km to compare RO and ionosonde results, showing that the error growth with distance is clearly larger for the classic approach than for the SM and that the advantage of the SM with respect to the classic approach relies on this capability to mitigate the LoC error when comparing with ionosonde measurements. However, Habarulema et al. (2014) have shown that there is an optimum window size (about 4° x 4° longitude/latitude around the ionosonde location) that minimizes the LoC error. Thus, it is not clear if the differences in the LoC error between different studies could explain the small improvement shown by the TEC-aided method reported in Yue et al. (2013) after using a narrow window or, alternatively, if the small improvement is due to the accuracy of the GIMs used in their study.

Another issue raised by Yue et al. (2013) was that their TEC-aided Abel inversion method showed practically no improvement with regard to the classic approach when comparing between large-scale ionospheric features of the $N_e$ retrievals, particularly the so-called artificial plasma caves observed in monthly averaged electron density values at low-latitude regions. On the contrary, the methodologies based on $N_{mF2}$ or $N_e$ global maps give rise to a significant reduction of the artificial plasma caves when compared with the retrievals from the classic approach (Chou et al., 2017; Pedatella et al., 2015a). Hence, despite being a promising methodology to improve the classic approach, TEC-aided methods have been challenged as providing a limited or negligible improvement.

The performance of TEC-aided methods could be determined by the accuracy of the TEC GIM used in the derivation of the EDPs. However, this assessment can hardly be quantified unless TEC GIMs of significantly different performance are used. In this respect, TEC-aided methods have used the TEC derived from IGS GIMs that consider the ionosphere distributed in a single layer at the height of 450 km. But, Hernández-Pajares et al. (2002) introduced an ionospheric model with an additional layer to account for the ionospheric delays in radio signals occurring at larger heights. The errors of the ionospheric delays predicted by those two-layer GIMs, known as the Group of Astronomy and Geomatics (GAGE) GIMs, were shown in Rovira-Garcia, Juan, Sanz, González-Casado, Ibáñez-Segura (2016) to be significantly smaller than the corresponding errors from the IGS GIMs.

On the other hand, according to González-Casado et al. (2013), the EDP retrievals from ROs depend only on the electron content (EC) below the LEO so that the EC above the LEO only affects the $N_e$ retrievals at the altitudes near the LEO height. However, this finding implies that the TEC-aided inversion could be affected by some mismodeling because TEC is used instead of EC below the LEO satellite height to describe the
horizontal gradient in the electron density around the tangent points of the RO. This would explain the worse performance shown by TEC-aided methods compared with methods that use $N_{m}F_{2}$ or $N_{e}$ global maps based on previously calculated EDPs from the classic approach. The use of the two-layer GAGE GIMs provides, for the first time, the possibility to address the impact of that mismodeling in the EDP retrievals.

Taking into account the previous list of considerations and the fact that TEC-aided Abel inversion is the only method that currently approaches to a near-real-time derivation of accurate EDPs, an assessment of the improvement of RO retrievals as a function of the accuracy of different TEC GIMs is strongly motivated. The present research examines the improvement achieved by the TEC-aided SM with regard to the classic approach and the difference between the improvements obtained by means of two-layer GAGE GIMs and single-layer IGS GIMs, where the greater accuracy of GAGE GIMs should provide better EDP retrievals. For this purpose, mainly global comparisons are analyzed. Regional comparisons focused in the European midlatitude and the South American low-latitude continental regions are considered to illustrate that, in regions where a dense network of ground receivers has been used to calculate the GIMs, the performance of EDP retrievals derived with the SM is improved. The analysis is focused in the $F$ region peak and the lower altitudes in the $E$ region, using in each case the same data set, the same criteria to select collocated measurements when comparing with reference ionosonde data, and an adequate metric to calibrate the improvement attained. Additionally, we review the fundamentals of the TEC-aided Abel inversion, addressing a way for overcoming the mismodeling incurred when TEC is used instead of the EC below the LEO satellite altitude.

2. Methodology

The basic concepts and the equations used by the SM are summarized in this section. The SM assumes that the electron density, depending on the height, $h$; longitude, $\lambda$; and latitude, $\phi$, for a given time, can be modeled as the product of two functions: one depending only on the altitude, $F(h)$, referred to hereafter as shape function, and another depending on the longitude and latitude. In its original formulation, the SM uses a horizontal map of TEC to represent the electron density as

$$N_e(\lambda, \phi, h) = TEC(\lambda, \phi) \cdot F(h)$$  \hspace{1cm} (1)

From equation (1) and assuming that the TEC can be known, for example, from an IGS GIM, the EDP can be determined by calculating the shape function $F(h)$. Notice that one can obtain a vertical EDP multiplying $F(h)$ times the TEC at any location in the region covered by the RO.

However, the horizontal map could be associated with any other magnitude linked to the EDP instead of TEC. For instance, one could use a map for $N_{m}F_{2}$ to express the electron density as

$$N_e(\lambda, \phi, h) = NmF2(\lambda, \phi) \cdot f(h)$$ \hspace{1cm} (2)

where $f(h)$ is also a shape function describing the vertical variation of the EDP. Then, integrating this equation over the altitude, one can obtain the vertical TEC at any horizontal coordinates ($\lambda$, $\phi$) as

$$TEC(\lambda, \phi) = NmF2(\lambda, \phi) \int_0^\infty f(h) dh$$ \hspace{1cm} (3)

where the integral on the right side is the so-called slab thickness parameter, which only depends on the vertical shape of the profile. Therefore, taking into account that the SM implicitly assumes a constant slab thickness, the use of a TEC map, as in Hernández-Pajares et al. (2000), or a horizontal map of $N_{m}F_{2}$ values as, for instance, in the iterative method of Pedatella et al. (2015a) is essentially equivalent.

Assuming one of the relationships (1) or (2), the EDP can be retrieved by means of the geometry-free combination ($L_{GF}$) of GPS measurements collected by a dedicated receiver on board of a LEO satellite during an RO and using the following equation:

$$L_{GF} = L_1 - L_2 = \omega \cdot STEC + b$$ \hspace{1cm} (4)

where $L_1$ and $L_2$ are the GNSS carrier phase measurements at two different frequencies and STEC is the slant TEC, the integrated electron density along the raypath between the GPS and the LEO satellites, defined as.
Further, α = 0.105 m/TECU is a conversion factor between STEC units and the L_{GF} units, and finally, b is a constant bias for a continuous arc of measurements during the RO. As it is shown in González-Casado et al. (2013), the EDP retrievals do not depend on the values that affect all the measurements of an RO equally, such as the EC above the LEO or the constant b. The impact of different choices for those values is negligible, since they can be removed by subtracting from all measurements a common reference value of L_{GF}, which can be selected, for example, from the measurement taken when the GPS satellite is at the LEO satellite horizon during the RO. Therefore, taking into account the geometry of a LEO-GPS observation and assuming an onion-skin model as illustrated in Figure 1 (see also Hernandez-Pajares et al. 2000), the STEC associated to an L_{GF} measurement with impact parameter p_i, can be written according to the SM in a discretized manner as follows:

\[
\text{STEC}(p_i) = 2\alpha \sum_{j=1}^{N} F(p_i) = 2\alpha \sum_{j=1}^{N} \left[ \int_{\text{TEC}(\lambda_j, \varphi_j)}^{\text{TEC}(\lambda_{j+1}, \varphi_{j+1})} F(p_j) \right] \]

where \(p_i\) is the geocentric distance of the tangent point \(P_i\) of the \(i\)th LEO-GPS ray; \(F(p_j)\) is the shape function value at the altitude \(p_j\); \(l_j\) is the distance between points \(P_j\) and \(P_{j+1}\), which is equal to the distance between points \(P_{j-i}\) and \(P_{j+i+1}\) and \(\text{TEC}(\lambda_j, \varphi_j)\) is the TEC at point \(P_j\) with longitude \(\lambda_j\) and latitude \(\varphi_j\).

Starting from \(R_i\), the spherical shell where the observation tangent point coincides with the LEO position, the value of the shape function \(F\) can be derived for the tangent point \(P_{i+1}\) following González-Casado et al. (2013). From this point, equation (6) can be solved in a recursive way obtaining, in each step, the value of \(F\) for each tangent point \(P_j\).

According to equation (6), if one considers a uniform TEC, so no horizontal gradient exists, a spherical symmetry relationship used by the classic approach is obtained. Therefore, when the external TEC GIM is highly smoothed, like in poorly sounded regions, the retrieved EDP will be very similar to the retrievals obtained using the classic approach.

Moreover, using equation (1), each term of the right side of equation (6) can be written as

\[
l_j \left[ \text{TEC}(\lambda_j, \varphi_j) + \text{TEC}(\lambda_{j+1}, \varphi_{j+1}) \right] F(p_j) = l_j \frac{\text{TEC}(\lambda_j, \varphi_j) + \text{TEC}(\lambda_{j+1}, \varphi_{j+1})}{\text{TEC}(\lambda_j, \varphi_j)} N_e(p_j)
\]

where the fraction on the right side of equation (7) is equivalent to an asymmetry factor. Consequently, the techniques using asymmetry factors (Chou et al., 2017; Pedatella et al., 2015a; Tulasi Ram et al., 2016) are equivalent to the SM, the only relevant difference being the global map used to calculate those factors as shown by equations (1) and (2). However, the advantage of using TEC maps to aid the RO inversion is that the corresponding GIM can be calculated using only data from a full day, having a significantly lower latency than the maps based in the EDP retrievals from the classic approach, that usually require about one month of data for a sufficient worldwide coverage. Additionally, there is an important difference in the results obtained from the SM with regard to the other methodologies previously referred. The strategy used by the SM initially calculates a shape function \(F(p_j)\) that only depends on height, and the final vertical EDP is calculated using equation (1) with the TEC at a given location covered by the RO. On the contrary, the methods based on asymmetry factors can only retrieve an oblique EDP, because the tangent points used to calculate the \(N_e(p_j)\) values are not aligned vertically. Hence, for a given RO, large differences can be found between the peak electron density derived from an oblique EDP and the vertical profile from the SM.

### 2.1. A More Precise Estimate of Horizontal Gradients Below the LEO Satellite

One difference between the TEC derived from GLMs and the TEC obtained from previously determined EDPs, as in the case of iterative methods, is that the latter consider the EC below the LEO height, \(E_{C,\text{LEO}}\), while the TEC from GLMs includes also the EC above the LEO. From the retrieved EDP, one can calculate \(E_{C,\text{LEO}}\) by performing the following integral:
As was shown by González-Casado et al. (2013), the EDP retrievals from an RO are essentially a representation of the EC below the LEO satellite orbit, while the EC above the LEO only affects the values of the retrieved EDP near to the LEO satellite altitude. Nevertheless, the EC over the LEO satellite orbit is a substantial fraction of the TEC, typically larger than 20–30% (González-Casado et al., 2015), which may be a mismodeling of the SM with regard to iterative methods. Thus, in order to apply self-consistently the SM technique, the TEC values used in equation (6) should be replaced by the values of $E_{C,LEO}$, which better describe the true horizontal gradients in the region below the LEO satellite orbit. This mismodeling would be larger for LEO satellites at a lower altitude like, for example, the CHAMP satellite (Jakowski, 2005). In order to address the impact of that mismodeling on the final EDP, one can use the two-layer GAGE GIMs to find an approximate value of $E_{C,LEO}$ as the sum of the EC contributions from the bottom ionospheric layer of the GIM, $E_{C, bottom}$, plus some unknown fraction $\gamma$ of the EC from the top layer, $E_{C, top}$.

$$E_{C,LEO}(\lambda, \phi) = E_{C, bottom}(\lambda, \phi) + \gamma E_{C, top}(\lambda, \phi)$$

The value of $\beta$ from equation (9), estimated after inverting the RO using the TEC = $E_{C, bottom} + E_{C, top}$ from the GAGE GIMs, can be used to derive the parameter $\gamma$ by means of the following equation:

$$\gamma = \frac{\beta \cdot TEC - E_{C, bottom}}{E_{C, top}}$$

The $E_{C,LEO}$ values obtained from equation (10) can be used to replace the original TEC used in equation (6), yielding a new derivation of the EDP that can be compared with the original EDP to assess the magnitude of the mismodeling. This would be a similar approach as used by iterative methods (Pedatella et al., 2015a). However, from equation (6), if the ratio between the $E_{C,LEO}$ and the original TEC from the GIM is nearly constant for all the locations covered by the RO, then differences will be found in the results obtained for the shape function $f(h)$ but not in the corresponding EDP. Finally, note that computing the $E_{C,LEO}$ in this way is more efficient than in the case of iterative methods, since only the parameter $\gamma$ must be calculated while iterative methods require the calculation, after the first iteration, of a grid map of $N_p F_2$.

3. Data, Tests, and Metric Used for Comparisons

This section describes the different quantities and statistical parameters used to assess the performances of the different techniques considered in the present study for electron density retrieval from RO measurements. These techniques are spherical symmetry Abel inversion or classic approach, SM aided by the TEC extracted from the IGS GIMs and SM aided by the TEC extracted from the GAGE GIMs, which also use the measurements from IGS ground receivers. At the end of this section, the data sets considered for the different comparisons are also described.

3.1. Description of the Tests

Two target quantities are considered to assess the performance of the RO inversion methods studied. The first one measures the error accumulated by the use of the recursive strategy, commonly applied in equation (6) to retrieve the EDP. Due to this recursive procedure, the errors in the electron density retrieval propagate downward in altitude, as it has been reported in several works. Therefore, the worst estimated values will correspond to the retrievals at the lowest heights. However, for altitudes below 100 km in the $D$ region, very low ionization level is expected, with mean electron densities smaller than about $10^4$ e$^-$/cm$^3$ (Hunsucker & Hargreaves, 2003). Integrating that electron density in the altitude range from 80 to 100 km

$$E_{C,100} = \int_{80}^{100} N_e \, dh$$

yields an electron content lower than 0.02 TECU. Meteorite showers, penetration of high-energy particles during strong geomagnetic storms, and tropospheric lightning can increase the electron density in the $D$
region. However, in normal conditions, \( EC_{100} \) will keep bounded most of the time by a very low value of nearly 0.02 TECU, close to zero (Mende et al., 2005; Osepian et al., 2009). Hence, if large deviations from that reference value are frequently obtained from an EDP retrieval method, they should be indicative of worse performance in comparison with another method achieving more frequent near-zero values of \( EC_{100} \).

Note that the testing based on \( EC_{100} \) does not rely on the comparison with reference ground measurements at a specific location, and hence, it is not affected by the LoC effect as in the case of comparing collocated ROs with ionosonde measurements. The \( EC_{100} \) test can be performed for all locations, over continents or oceans, where RO observations are available. Notice also that this test is different from other tests that analyze the electron density retrievals at low altitudes, where the improvement is assessed by quantifying the reduction of the artificial plasma caves using the results obtained with the classic approach as reference values (Pedatella et al., 2015a; Yue et al., 2013). While the test proposed in the present study compares the RO retrieved \( EC_{100} \) with a reference value according to observations.

The second target quantity that will be analyzed to test the performance of the retrieved EDPs is the \( F_2 \)-layer critical frequency, \( f_{OF_2} \), measured in Hz, which can be derived from the peak electron density through the relationship \( NmF_2 = (f_{OF_2}/8.98)^2 \), with \( NmF_2 \) given in \( e^- m^-3 \). The \( f_{OF_2} \) values derived from EDPs will be compared with ionosonde measurements that will be taken as reference values to check the quality of the RO inversion. In this way, one can assess the error of the RO results by calculating the distribution of the relative difference with respect to ionosonde measurements:

\[
\Delta r_{f_{OF_2}} = \frac{f_{OF_2, RO} - f_{OF_2, Ionosonde}}{f_{OF_2, Ionosonde}}
\]  

Since early studies, this relative difference has been used as a metric for assessing the accuracy of \( f_{OF_2} \) retrievals (e.g., Hajj & Romans, 1998; Hernández-Pajares et al., 2000; Schreiner et al., 1999) and more recently in Pedatella et al. (2015b) or Habarulema and Carelse (2016).

Note that due to the quadratic relationship between critical frequency and peak electron density, when the error is small one should expect that, typically, the relative difference in \( NmF_2 \) is nearly 2 times \( \Delta r_{f_{OF_2}} \). Indeed, this has been confirmed by the error distributions obtained in this research. On the other hand, in the SM all the points where the RO is inverted share the same shape function. In this sense, because the altitude of the peak electron density, \( h_{mF_2} \), is a characteristic of the shape function, one will obtain similar values regardless whether the classic method or the SM method is applied. For those reasons, we only present the results corresponding to the relative error of \( f_{OF_2} \).

3.2. Metric

To avoid the contamination by extreme outliers in the distributions of \( EC_{100} \) and \( \Delta r_{f_{OF_2}} \), values outside the interval [−1, 1] have been excluded from the distributions. In the case of \( EC_{100} \), since it has been calculated in TECU, the values outside that interval will correspond to mean electron densities in the D region greater than \( 5 \times 10^5 \) \( e^- cm^-3 \), which is an unusually high value as will be illustrated by the results presented in section 4. In the case of \( \Delta r_{f_{OF_2}} \), values outside [−1, 1] correspond to a relative difference greater than 100% between RO and ionosonde measurements. In both cases, that preliminary filtering of the distributions excludes less than 1% of the original data set.

On the other hand, we will use the 68% and 95% percentiles of the distributions of the target quantities as metrics. Those percentiles approximately correspond to one and two standard deviations of a zero-mean Gaussian distribution, respectively. In previous studies, the root mean square of absolute or relative errors has been used to calibrate the improvement achieved by different methods. Although the root mean square is an adequate metric when the error distribution is close to a Gaussian, its value can be significantly affected if the distribution has long tails with a few outliers giving extremely large error values compared with the bulk of the error distribution. Instead, the percentiles provide an unambiguous statistical measure of the true shape of the error distribution. Moreover, in the presentation of the results, it will be shown that the distributions of the target quantities analyzed are clearly non-Gaussian.
From the value of the percentiles of the target quantity derived from the different methods considered, the percent of improvement, \( W \), of a given method “B” for EDP retrieval with regard to another method “A” can be quantified as follows:

\[
W = 100 \left( \frac{P_{68}(A) - P_{68}(B)}{P_{68}(A)} \right)
\]

\[\text{Equation (14)}\]

\( P_{68}(A) \) and \( P_{68}(B) \) being the corresponding 68% percentile of the target quantity derived after application of the methods A and B, respectively. Since lower values of the 68% percentile correspond to a more centrally concentrated distribution, they reflect a better performance of the corresponding method in a statistical sense. Thus, the \( W \) parameter measures the typical percentage of error reduction, as reflected by the 68% percentile of the error distribution, which is achieved by the method having better performance in the comparison.

3.3. Data Set

As the data set for the comparisons, we have used ROs observed by the FORMOSAT-3/COSMIC constellation of LEO satellites. This constellation consists of a set of six satellites taking nearly worldwide distributed measurements that sound the ionosphere during RO of GPS satellites. The data of observed ROs during 180 days of 2014, evenly distributed, have been collected from the Cosmic Data Analysis and Archive Center and processed to retrieve nearly \( 1.5 \times 10^5 \) EDPs for each of the different methods analyzed in this research. The altitude of the COSMIC satellite orbits is around 800 km; the maximum altitude reached by the EDPs used in this research goes from 740 to 860 km. All the profiles were retrieved down to a minimum altitude of 80 km when possible, selecting profiles according to an \( F_2 \)-layer peak altitude in the range from 150 to 450 km, to include in the data set the profiles retrieved from different latitudes and during different seasons of the year, which may have different \( F_2 \)-layer peak altitudes. On the other hand, in order to perform a reliable comparison of the quality of the retrievals calculated with different TEC maps, only the RO measurements located in the longitude range \([0°, 360°]\) have been considered since; according to Rovira-Garcia, Juan, Sanz, González-Casado, Ibáñez-Segura (2016), the GAGE GIMs were computed using permanent receivers located within that longitude range. Regarding the \( EC_{100} \) testing, the ROs selected did not have any restriction in location except for the longitudinal range mentioned before. Note that the RO retrieved \( EC_{100} \) is compared with an observed value, close to zero in TECU, which is assumed to be the same worldwide (see section 3.1). For the \( \Delta f_2 \) test, \( F_2 \)-peak critical frequency data were download from the Digital Ionogram DataBase of Global Ionospheric Radio Observatory (Reinisch & Galkin, 2011). These \( f_2F_2 \) values were derived from ionograms manually or automatically scaled from 48 ionosondes, in the locations depicted in Figure 2, during 2014. The autoscaling confidence score (CS) value provided by the database (ranging between 0 and 100) was used to select reliable ionograms when automatically scaled. Essentially all selected measurements had CS greater than 50, with 70% of measurements having CS > 80.

When comparing ionosonde and RO results, only collocated ROs were selected. Specifically, we have considered the subset of ROs yielding an \( F_2 \) peak located inside the region with a longitude difference smaller than \( \Delta \lambda = 8° \) and a latitude difference below \( \Delta \varphi = 5° \) with respect to the ionosonde location. Moreover, comparisons were done only when the time difference between RO and ionosonde observations was less than 0.5 hr. Figure 2 also shows the locations of the \( F_2 \)-layer peaks from ROs observed within that spatial and temporal window. The effects of the window size used to collocate ROs have been considered, repeating the comparisons using a reduced window with \( \Delta \lambda = 5° \) and \( \Delta \varphi = 2.5° \).

4. Results

One of the goals of the present study is to assess if the performances of the TEC-aided retrieval methods depend on the accuracy of the TEC values. To this end, we focus on worldwide results including regions poorly or not sampled by ground receivers in the oceans. Hence, to illustrate that the EDP retrievals are improved in continental regions where the GIMs have been calculated using a dense network of ground IGS receivers, we also present the results for two continental regions at different latitudes: the South
American sector approximately ±20° around the magnetic equator and European locations with geographic latitudes in [30°, 60°]. A dense network of ground receivers provides greater accuracy in TEC predictions, as shown by the analysis of the positioning error in radio navigation after using IGS and GAGE GIMs (Orús, 2017; Rovira-Garcia, Juan, Sanz, González-Casado, Bertran, 2016). Therefore, the regional results will illustrate the performance of the SM for two specific regions, in middle and low latitudes respectively, where the TEC is well sounded by the GIMs considered in the present study.

4.1. EC100 Test

Figure 3 depicts the histogram of the EC100 values obtained from the three methods considered, the classic approach, SM aided by IGS GIMs and SM aided by GAGE GIMs, using a bin size equal to 0.02. The bottom panel shows global results, including locations over oceans and continents where ROs have been observed, while in the top panel, one can see the results for ROs observed over Europe. The mean value of the global EC100 distribution from the classic approach is about 0.04, nearly a factor of 2 greater than the corresponding mean values obtained with the SM, that are slightly smaller than 0.02. A similar difference between the mean values of the classic approach and the SM is also obtained in the regional distributions for Europe and South America.

Table 1 shows the 68% and 95% percentiles of the distributions of the EC100 parameter from the different methods, globally and for the European and South American regions. The total number of ROs analyzed in each region is given in the second column. Figure 3 and the percentiles presented in Table 1 illustrate that it is very frequent to find values of EC100 considerably larger than 0.02 TECU, the expected upper limit value in the D region. On the other hand, comparing the results from the different methods, one can observe a clear reduction in the 68% percentile from the SM with regard to the classic approach, indicating an improvement in the low-altitude retrievals. Specifically, SM with IGS GIMs improves the results of the classic approach, according to parameter W defined in equation (14), by nearly 39% in Europe, 27% in South America, and 32% globally, while the improvement using the SM with the GAGE GIMs is 45% in Europe, 34% in South America, and nearly 32% globally. Therefore, comparing the results of the SM using different GIMs, the GAGE GIMs are 10% better in Europe and South America than the IGS GIMs, but globally, no improvement is observed. This last result is not surprising since the global results include poorly sampled regions over the oceans, where both types of GIMs have lower accuracy. On the other hand, the percentage of improvement observed in the low-latitude South American region is smaller than in the European region, for both types of GIMs, IGS and GAGE. Even though, the improvement in the low-latitude region is still very large with regard to the classic approach results and it is very similar to the improvement attained globally. Clearly, the existence of large ionospheric gradients in low-latitude regions affects the performance of GIMs, which have typically lower accuracy than in midlatitude regions (Rovira-Garcia, Juan, Sanz, González-Casado, Ibáñez-Segura, 2016).

Figures 4a–4c depict, respectively, the mean EC100 at different geomagnetic latitudes from the classic approach, the SM aided by GAGE GIMS and their difference, calculated for March 2014 and for two local time
Figure 4. Mean EC\textsubscript{100} for March 2014 between 0:00 and 2:00 LT (blue solid lines) and between 13:00 and 15:00 LT (red solid lines) from (a) the classic approach, (b) the SM with GAGE GIMs, and (c) the difference between Figures 4a and 4b. Zonal mean electron density distribution for March 2014 between 0:00 and 2:00 LT for (d) the classic approach, (e) the SM with GAGE GIMs, and (f) the difference between Figures 4d and 4e. (g–i) The same as in Figures 4d–4f but for results between 13:00 and 15:00 LT. Black contour lines represent zero electron density.
Compared with the very small value expected for $E_C^{100}$, Figure 4a shows clear overestimations in midlatitude regions, while significant underestimations are seen for low latitudes. Indeed, as it is reported in Yue et al. (2013) and Pedatella et al. (2015a), the classic approach cannot account for the large gradients in the low-latitude regions and produces a clear underestimation in the equatorial region and an overestimation in the surrounding midlatitude regions as observed in Figure 4a. As seen in Figure 4b, the deviations from the small value of a few hundredths of TECU expected for $E_C^{100}$ are clearly mitigated in the retrievals from the SM aided by the GAGE GIMs. The reduction of these deviations are quantified in Figure 4c, where one can see a maximum reduction of nearly 0.2 TECU at midnight and close to 0.4 TECU at noon, corresponding to a reduction of the deviations of the mean electron density equal to $10^5$ and $2 \times 10^5$ e$^{-}$ cm$^{-3}$, respectively, in the altitude region from 80 to 100 km.

The average electron density distribution for midnight and noon periods during March 2014 is presented in the middle and bottom rows of Figure 4. In agreement with the $E_C^{100}$ results shown in the top row, the artificial plasma caves around the equator observed in the EDP retrievals from the classic approach (Figures 4d and 4g for midnight and noon, respectively) are substantially mitigated when using the SM (Figures 4e and 4h for midnight and noon, respectively). The electron density values derived for the classic approach are consistent with the values obtained in previous studies (Pedatella et al., 2015a) for the same LT intervals during a period with similar ionospheric activity as March 2014 due to the semiannual anomaly that occurs during the equinoxes. However, in Figure 4f (the difference between Figures 4d and 4e), one can observe that the difference between the classic approach and the SM retrievals is close to $2 \times 10^5$ e$^{-}$ cm$^{-3}$ for altitudes below 200 km in the low-latitude region during midnight. This is 1 order of magnitude larger than the corresponding values derived in Pedatella et al. (2015a) using their iterative method. On the other hand, during noon, the corresponding reduction of the artificial plasma caves observed in Figure 4i (the difference between Figures 4g and 4h) is a factor of 4 to 8 larger than reported in that previous study for altitudes around and below 300 km. Hence, we conclude that the SM using more precise two-layer TEC maps and nonoblique EDPs clearly improves the classic approach, providing a more accurate description of the large electron density gradients that characterize the equatorial region during the equinox of 2014.

### 4.2. Comparison With Ionosonde Measurements

Figure 5 shows the histogram of relative differences $\Delta r_{foF2}$, see equation (13), between RO inversion results and the ionosonde measurements at the global scale, from the different methods considered and using a bin size equal to 0.01. From this figure, one can see that there is a significant improvement from classic approach to SM inversion using IGS GIMs, while this improvement is even greater when using GAGE GIMs. Table 2 shows the percentiles of the distributions of $\Delta r_{foF2}$ globally and for the European and South American regions. The second column shows the total number of ionosonde-RO pairs. In Europe, the 12 ionosondes in Figure 2 within the longitudinal range $[-10^\circ, 60^\circ]$ were considered for the comparisons. In South America, in order to collect a sufficient number of ROs for the comparisons, from the six ionosondes in the low-latitude region, two more ionosondes were considered, one in the Caribbean region and the other in the Atlantic Ocean (Ascension island), both located within $\pm 20^\circ$ around the equator. Note that, as reported in previous works (Habarulema & Ssessanga, 2017), one RO can be compared with two different ionosondes when they are located close enough.

**Table 2**

<table>
<thead>
<tr>
<th>Region</th>
<th>Number (×$10^3$)</th>
<th>Classic 68%</th>
<th>95%</th>
<th>IGS GIM 68%</th>
<th>95%</th>
<th>GAGE GIM 68%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>17.7</td>
<td>0.106</td>
<td>0.356</td>
<td>0.077</td>
<td>0.288</td>
<td>0.072</td>
<td>0.274</td>
</tr>
<tr>
<td>Europe</td>
<td>6.0</td>
<td>0.092</td>
<td>0.288</td>
<td>0.063</td>
<td>0.202</td>
<td>0.056</td>
<td>0.180</td>
</tr>
<tr>
<td>S.A. low latitude</td>
<td>1.1</td>
<td>0.128</td>
<td>0.401</td>
<td>0.104</td>
<td>0.390</td>
<td>0.100</td>
<td>0.345</td>
</tr>
</tbody>
</table>
Focusing in the results for the 68% percentile in Table 2, the improvement achieved by the SM based on IGS GIMs in comparison with the classic approach is 32% in Europe, 19% in South America, and 27% globally. In the case of the GAGE GIMs, the corresponding improvements increase to 39% in Europe, 22% in South America, and 32% globally. Thus, the use of a GAGE GIM improves the results, with respect to the use of an IGS GIM, by nearly 11% in Europe, 4% in South America, and 6% globally. The percentage of improvement of the SM with regard to the classic approach in the low-latitude South American region is, in general, smaller than in the European region and globally. The comparison between the results of the SM using different GIMs gives also less improvement in the South American region than globally and in Europe. This is clearly a consequence of the larger errors found in the low-latitude region according to the 68% and 95% percentiles shown in Table 2.

On the other hand, the smallest values of the 95% percentile, reflecting the tails and overall shape of the distributions, are obtained for the SM in both Tables 1 and 2. Note that, in general, the 95% percentiles are more than 3 times larger than the 68% percentiles, which reflects the relevance of the tails in the error distributions that clearly deviate from a Gaussian shape.

Finally, the effect of a narrower window around ionosondes to collocate the ROs has been considered, repeating the previous calculations using a window with $\Delta \lambda = 5^\circ$ and $\Delta \phi = 2.5^\circ$, which were the same values used in the study of Yue et al. (2013). Since the SM better manages the LoC, the reduction of the region around ionosondes used to collocate ROs benefits to the classic approach. Compared to the results presented in Table 2, the 68% percentile values decrease about a 12% for the classic approach, a 9% for SM using IGS GIMs and a 6% for SM using GAGE GIMs. However, this reduces the improvements with regard to the classic approach previously derived from Table 2 in less than 4% in all the cases.

### 4.3. Analysis of the Mismodeling in TEC-Aided Methods

The $\beta$ parameter defined in equation (9) provides a measure of the ratio between the electron content below the LEO satellite orbit and the TEC. As explained in section 2.1, the TEC values used in equation (6) should be replaced by the $EC_{LEO}$ values if one applies self-consistently the SM. Thus, according to equation (8), the mismodeling affecting the SM should be smaller when $\beta$ is closer to unity. Figure 6 depicts the histograms of the distribution of $\beta$ values when the EDPs are derived using the SM aided by the IGS (in red) or GAGE GIMs (in blue). In both cases, in agreement with previous estimates by González-Casado et al. (2015), the values of $\beta$ are clearly smaller than 1.

One can calculate the $\gamma$ parameter for each RO following equation (11) and the corresponding $EC_{LEO}$ can be derived using the GAGE GIM and equation (10). Substituting the original TEC by the new calculated $EC_{LEO}$ in equation (6), a new retrieval of the EDP can be obtained. The cyan curve, shown in Figure 6, corresponds to the $\beta$ histogram derived from those new EDPs using equation (9). As one can see, the new histogram is mostly peaked around 1; this means that the EC that has been used in the RO inversion essentially coincides with $EC_{LEO}$ in a similar way as in the iterative methods. Hence, the most reliable EDP retrievals can be selected by considering the $\beta$ values closest to unity in that new histogram. Table 3 shows a quantitative assessment of the 68% percentile of the $f_oF_2$ distributions of relative error in the European region after selecting the ROs yielding a value of $\beta$ in a restricted narrow range around unity from the cyan curve in Figure 6. When filtering the EDP retrievals in that way, one can see that all the techniques improve their performance with regard to the results presented in Table 2 for Europe. Particularly, when the range of $\beta$ values is very close to unity (last row in Table 3), the improvement is nearly a 10% with regard to the results obtained without any $\beta$ filtering (second row in Table 2).

### Table 3

<table>
<thead>
<tr>
<th>Filter Range</th>
<th>Number ($\times 10^3$)</th>
<th>68% Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Classic</td>
</tr>
<tr>
<td>0.95 ≤ $\beta$ ≤ 1.05</td>
<td>5.6</td>
<td>0.087</td>
</tr>
<tr>
<td>0.995 ≤ $\beta$ ≤ 1.005</td>
<td>5.2</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Figure 6. Distribution of $\beta$ values obtained when using different GIMs to aid the SM: IGS GIM (red), GAGE GIM (blue), and $EC_{LEO}$ calculated after the derivation of $\gamma$ parameter for each RO (cyan).
On the other hand, the relative improvements between the different methods in Table 2 are maintained in Table 3. This finding implies that filtering the EDP retrievals according to a value of $\beta$ near to unity is a strategy providing a quality control of the mismodeling of the SM by selecting a more accurate set of profiles than the original one used in Table 2.

However, there is little difference between the error distributions of $f_{0}F_{2}$ values obtained with the SM using the $E_{\text{CLEO}}$ or the original TEC from GAGE GIMs, which explains the similar 68% percentile seen in Table 3. This result strongly indicates that $E_{\text{CLEO}}$ and TEC values used in equation (6) are, in most cases, nearly proportional at all tangent points sampled by a given RO. In turn, this finding implies that, in terms of the retrieved $f_{0}F_{2}$, there is only a small mismodeling affecting the final EDP retrieval when using TEC from GAGE GIMs to describe the horizontal gradients in the ionosphere electron density.

Given the small values of the 68% percentile achieved by the SM when compared with ionosonde measurements in Europe, we have considered two closely located ionosondes in that region, Fairford (51.7°N, 1.5°W) and Chilton (51.5°N, 0.6°W), to see if the observed differences between their $f_{0}F_{2}$ measurements are similar to the errors achieved by the SM. The baseline of these two ionosondes is less than 70 km, which is clearly smaller than the window used in the comparison of ROs with ionosondes. Figure 7 shows the cumulative distribution function of the relative differences between the Chilton ionosonde measurements and the Fairford ones. In the same figure we show the corresponding cumulative distribution functions of the relative difference, with regard to Fairford measurements, achieved by the different methods previously considered. These comparisons are done for a total number of 510 collocated ROs. From Figure 7, one can clearly observe that both GAGE GIM methods and ionosonde observations achieve relative errors smaller than 7% in nearly 82% of the cases. Small errors ($0.05 \leq \Delta f_{0}F_{2} \leq 0.05$) are found for nearly 77% of cases for ionosonde comparisons, 70% for SM using GAGE or $E_{\text{CLEO}}$, 62% for SM using IGS GIMs, and 50% for the classic approach. The 68% percentile values from the cumulative distribution functions shown in Figure 7 are 0.035 for the comparison between ionosondes, 0.049 and 0.047, respectively, for SM using GAGE or $E_{\text{CLEO}}$, 0.059 for SM using IGS GIMs, and 0.087 for the classic approach. The percentages of improvement between different methods are similar to the ones inferred from Tables 2 and 3 in the European region.

5. Conclusions

The main goal of this research is to show the impact of the accuracy of the GIMs used by the SM Abel inversion technique on the quality of the RO retrieved EDP. This quality has been assessed globally and by considering two specific regions where the performance of the GIMs is expected to be different from global performance. Previously published studies (Aragon-Angel et al., 2010; García-Fernández et al., 2003, 2005; Yue et al., 2013) suggested that the improvement of the SM, with regard to the classic approach, is small and/or essentially due to the LoC error, which is substantially mitigated when the SM is used. However, our results do not agree with this interpretation.

The present study has demonstrated that the SM provides significantly greater accuracy than the classic approach based on the spherical symmetry assumption. Moreover, it has been shown that the use of more accurate GIMs gives rise to an additional improvement in the accuracy of the RO retrievals. First, through the $E_{\text{C100}}$ test, which is not affected by the LoC error, it has been shown that, according to the 68% percentile of the error distributions, the results using SM aided by GAGE GIMs can improve by 30% worldwide, 45% over Europe, and 34% over low latitudes in South America the results obtained using the classic approach. In the continental regions analyzed, where GIMs are well sampled by dense networks of ground receivers, the two-layer GAGE GIMs provide a 10% of improvement in the RO retrievals in comparison with the results using single-layer IGS GIMs. Moreover, the SM substantially mitigates the artificial plasma caves produced by the
large ionospheric gradients in the equatorial region when the classic approach is used. Second, by comparing with ionosonde measurements, similar improvements as those with the $EC_{100}$ test are obtained. Indeed, we have found relative errors using the SM that represent a 32% improvement worldwide and a 39% in Europe with respect to the results using the classic technique, while the corresponding improvement in the low-latitude region is 22%. RO retrieved critical frequencies from two-layer GAGE GIMs are improved around 5% in general, but 11% in Europe, with regard to the retrievals from single-layer GIMs.

Finally, introducing a quality criterion based on selecting the ROs yielding a value of $\beta$ around unity, it has been shown that the SM aided by GAGE GIMs can achieve relative errors, at the 68% percentile, or nearly 5% in the retrieved $f_{pF_2}$, similar to the errors obtained when comparing the results from two nearby ionosondes. Using the SM with GAGE GIMs in a well-sounded area like Europe, one can retrieve the peak electron density from ROs with an accuracy similar to that of ionosonde measurements, with errors smaller than 7% in nearly 82% of the cases during 2014.

Regions poorly sampled by ground receivers will not probably benefit from the significant improvement in the RO retrievals shown in Europe by the SM. However, in a second iteration, one can calculate new retrievals in order to increase the performance over the oceans by substituting the TEC value from the GIMs by the $EC_{LEO}$ calculated from equation (8), that is, the TEC obtained after integrating the EDP calculated in the first iteration. Note that this kind of iterative strategy is similar to the two-iteration process used in previous studies (e.g., Pedatella et al., 2015a). However, there is one important difference. With the SM aided by the daily two-layer GIM, one can perform the two iterations without any delay or latency between them. While the methods that use the classic approach in the first iteration need, before the second iteration, to calculate a map of the $N_mF_2$ or electron density, which presently requires several weeks of data in order to build a reliable map.

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